The Adoption of Non-Rival Inputs and Firm Scope*

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Abstract

Custom software is distinct from other types of capital because it is non-rival—once a firm invests in it, the software can be used simultaneously across its many establishments. Using confidential U.S. Census data, we document that while firms with more establishments are more likely to invest in custom software, they spend less on it as a share of total capital expenditures. We explain these empirical patterns by developing a model that incorporates the non-rivalry of software and the firm's choice of scope. Firms choose whether to adopt custom software, the intensity of their investment, and their scope, balancing the costs of managing multiple establishments against the increasing returns to scope from non-rival software. Calibrating the model with microdata, we show that improvements in custom software production account for a significant share of rising concentration and aggregate productivity growth. Abstracting from adoption and scope margins substantially understates these effects.

JEL Classification: D24, E22, O33
Keywords: Technology adoption, Non-rivalry, Concentration, Firm scope

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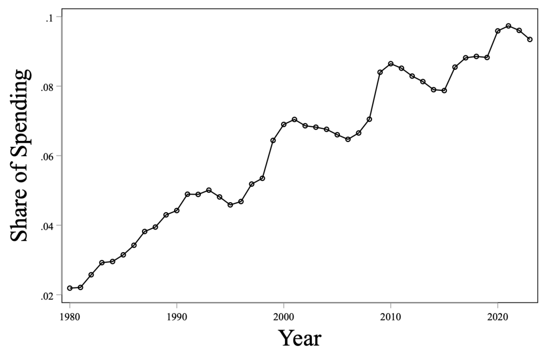
1 Introduction

Software investments have grown substantially over the past 40 years. A considerable portion of this growth is attributed to software that is unique to the firm, either developed in-house or customized by a vendor. Firm-level investments in such custom software surpassed 9 percent of U.S. nonresidential fixed investment in 2023, increasing from just 2 percent in 1980 (Figure 1). Unlike traditional capital, such as a cash register or forklift, which are limited to use at a single time and location, custom software is non-rival—a firm can use its custom software simultaneously across its many establishments. This distinct feature of software raises a number of questions about its impact on the boundaries of the firm and its investment decisions: How does the non-rivalry of custom software affect firms' adoption decisions, their allocation of investment between non-rival and traditional capital, and their choice of firm scope? What are the implications of technological improvements in custom software for concentration and aggregate productivity?

Despite its growing importance, studies addressing custom software and its implications are limited. Previous literature often lumps custom software with other rivalrous ICT investments or considers it a component of intangibles, which are difficult to measure. In this paper, we make three contributions. First, we use a novel dataset on firm-level investments in custom software to document how the adoption and intensity of investment in custom software vary with firm scope, measured by the number of establishments a firm operates. Second, we build a model in which firms choose whether or not to adopt a non-rival input and, if so, how much to invest. Firms choose their scope, balancing the cost of managing multiple establishments with the increasing returns to scope from the non-rivalrous investment. We show that the calibrated model can match the empirical patterns on custom software use. Third, we use the model to examine the aggregate impact of a decline in the rental rate of software, finding that advancements in the software sector can account for 20% of the rise in concentration and aggregate TFP growth. Importantly, these effects are attenuated in a model that abstracts from the choice of software adoption or intensity, underscoring the importance of matching our empirical findings when assessing the aggregate impact of software.

We start our analysis by documenting several motivating facts using the Annual Capital Expenditure Survey (ACES), a confidential dataset from the U.S. Census. The dataset provides detailed information on firm-level investment decisions across different capital categories, including custom software. We merge the ACES with the Revenue-Enhanced Longitudinal Business Database (LBD), which has information on the number of establishments a firm operates, our primary measure of firm scope, along with firm employment, payroll, and sales. We use the data to document three motivating facts on custom software investments across the distribution of firms. First, on

Figure 1: Custom Software Share of Total U.S. Non-Residential Fixed Investment



Notes: This figure shows the share of non-residential fixed investment that is accounted for by custom software, which we define as the sum of vendor-customized and own-account software. Source: BEA National Accounts Data.

the extensive margin, we show that the share of firms that invest in custom software increases with firm scope. Second, on the intensive margin, conditional on a firm having positive custom software investments (called "adopters" hereafter), the share of total investments devoted to custom software declines with firm scope. Third, among adopters, software and labor cost shares decline with firm scope, while the capital share rises, suggesting a non-homothetic production function.

Motivated by these empirical patterns, we build a model of heterogeneous firms that incorporates the non-rivalry of custom software and firms' choice of scope. In the model, firms choose between two production functions—one with only labor and capital and one that also incorporates custom software—along with the corresponding optimal choice of firm scope. Crucially, the software input is non-rival, allowing the firm to pay for the investment once and then use it costlessly at each establishment. Many papers model ICT or software as a fixed cost that lowers the marginal cost to the firm. Instead, we model software as a variable input that enters the production function similarly to capital and labor. However, unlike capital and labor, the software input can be shared across the firm's many establishments. As a result, the effective cost of software declines in firm scope. Firms choose the optimal scope by trading off the benefits from the reduction in the effective cost of software and the profits from an additional establishment against the span-of-control costs and within-firm cannibalization.

To quantify the impacts of increasing software investment, we calibrate the model using micro data from the ACES-LBD matched sample. The calibrated model is able to generate patterns consistent with the empirical facts. On the extensive margin, the effective cost of software is higher for firms with a small scope, and the largest firms will be more likely to opt for software adoption, consistent with our empirical finding that adoption of custom software is increasing in firm scope. On the intensive margin, because of the non-rivalry of the software input, the resulting production function for adopters is non-homothetic: the cost shares of software, capital, and labor vary endogenously with firm scope.[1] This feature matches our empirical findings and distinguishes the non-rival input mechanism from fixed-cost models of ICT, which do not naturally generate declining software and labor shares alongside a rising capital share as firms expand.

Finally, we use the model to examine the aggregate implications of technological changes in the production of custom software. To that end, we first calibrate our model to the current "software era" using data moments from 2018, the final year of our sample in the ACES. We then shock the model to the pre-software era in the late 1980s by reducing the productivity of the software-producing sector to match the increase in the rental rate of custom software compared with the present period. Through the lens of the model, the shock leads to a more-than-threefold increase in the adoption rate of custom software. The impacts are heterogeneous across firms. On the one hand, firms that do not adopt software contract their scope, driven by the general equilibrium increase in the wage. On the other hand, adopters, especially those that switch from non-adopter to adopter, experience increases in firm scope and market share. Even within the group of firms that adopt software, the impact of the shock is heterogeneous due to the non-homotheticity and the increasing returns to scope.

In aggregate, the shock to the rental rate of custom software can explain approximately 20 percent of the increase in aggregate custom software investment share. Moreover, the shock generates around 20 percent of the observed increase in the share of establishments owned by the top 1% of firms and the sales share of the top 1% of firms. Additionally, the model also generates a slight decrease in the aggregate labor share, though the magnitude of this decrease is small compared to the data.

The shock also generates substantial gains in aggregate TFP and labor productivity. To facilitate comparison with TFP measures in the data, we follow the BEA's methodology and compute TFP in the model using an aggregate Cobb–Douglas production function with software included in the capital stock. Through the lens of the model, the measure of aggregate TFP rises by 5.8%, accounting for roughly 20% of the TFP growth observed in the data. Beyond the direct contribution of productivity gains in the software sector, this increase—nearly double the benchmark in Hulten (1978)—is driven by: (1) reallocation toward the largest firms, (2) firms exploiting increasing returns to scope and moving down their unit-cost curves, and (3) efficiency gains from a decline in the price of the final good. Labor productivity increases by 8.3%, exceeding TFP growth due to capital deepening.

The aggregate implications hinge crucially on both the extensive-margin adoption of software and firm scope. We show in Section 6.3 that when the extensive margin is removed and all firms

[1] Homothetic production functions imply constant factor cost shares, independent of firm size or scope. The Cobb–Douglas and CES are standard examples.

are adopters, a decline in software prices reduces establishment and sales concentration while still delivering comparable TFP growth. In contrast, when firm scope is shut down, the model produces much smaller TFP gains, underscoring the role of scope in driving aggregate productivity growth. Thus, while software is non-rival, its use does not necessarily imply rising concentration or substantial TFP growth. These quantitative results hinge on our cross-sectional findings on how the adoption and use of custom software vary with firm scope.

Our model assumes that custom software is both non-rival and non-excludable within the firm. While all software is non-rival, there may be other factors that restrict a firm's ability to use its investment costlessly across multiple establishments. For instance, vendor-customized software can be subject to licensing arrangements, making it partially excludable. Moreover, software suitable for one establishment might require adjustments for another. In Section 6.4, we extend our model to account for the partial excludability and specificity of software and discuss the robustness of our main results.

While custom software in the model is non-excludable within the firm, it is excludable across firms. This is a key distinction between our model and endogenous growth models in which non-rival inputs, such as ideas, are at least partially non-excludable across firms, leading to economy-wide increasing returns to scale (e.g. Jones, 2005; Romer, 1990). In contrast, our model features increasing returns to scale within the firm but no spillovers across firms.[2]

The model we develop offers a comprehensive framework that integrates the adoption of non-rival inputs, the allocation of investment between non-rival and rival inputs, and the interaction with the firm's choice of scope. Although our analysis is centered on custom software as the non-rival input and the number of establishments as the measure of firm scope, the model is general and could be applied more broadly to other non-rival inputs such as brands, expertise, and patents. This paper focuses on custom software as a non-rival input for two key reasons. First is custom software's growing importance as a share of aggregate investment. Second, data on intangibles is scarce because they are hard to measure. In this case, we have reliable survey data on one of the best-measured intangibles, custom software. Similarly, one could think of many measures of firm scope, including the number of product lines or industries. We focus on the number of establishments as the primary measure of firm scope because previous literature has documented the importance of the growing number of establishments per firm for the rise in concentration (Hsieh and Rossi-Hansberg, 2023; Smith and Ocampo, 2020).

[2] Jones and Tonetti (2020) characterize the inefficiency that arises in an economy where a non-rival input, like data, is also partially excludable across firms.

Related literature. We make both empirical and theoretical contributions to the literature on intangible capital, such as brands, patents, managerial practices, expertise, data, and firm culture.[3] Intangibles are difficult to measure (Crouzet and Eberly, 2021; McGrattan, 2020; McGrattan and Prescott, 2010). We contribute by focusing on one form of intangibles for which we have high-quality firm-level data, custom software, which allows us to document patterns of custom software use across the distribution of firm scope and to analyze the growing importance of custom software as a form of investment.

Furthermore, building on a recent literature on non-rival inputs (Argente et al., 2021; Crouzet et al., 2022a,b; Ding, 2023; Kleinman, 2022), we develop a model of custom software as a variable input that is non-rival across the establishments of the firm. Relative to this literature, and motivated by our empirical findings, we add an extensive margin choice to adopt custom software, which is crucial in determining the distributional impacts of shocks across firms. Moreover, we show that the interaction between the choice of firm scope and the variable non-rival input provides a micro-foundation for a non-homothetic production function (Sato, 1977).[4] Non-homotheticity is central to explaining our empirical findings, particularly the fact that labor, capital, and software cost shares vary with firm scope, and to distinguishing between competing models of ICT and intangibles. A common approach in the literature treats ICT and intangible capital as a fixed cost that raises the firm's factor-neutral productivity (De Ridder, 2024; Hsieh and Rossi-Hansberg, 2023; Jiang, 2023; Mariscal et al., 2018; Rubinton, 2020). This type of framework cannot naturally account for the observed scope-dependent cost shares. By contrast, our model micro-founds a non-homothetic production function that generates declining labor and software shares and a rising capital share as firms expand their scope.

We also contribute to the extensive literature linking the rise of software, and ICT more broadly, to macroeconomic trends.[5] Closely related, Lashkari et al. (2024) finds that the fall in ICT prices can explain changes in concentration and the labor share in France and De Ridder (2024) finds that intangibles can account for the slowdown in productivity growth and the increase in market power. We enrich the literature by incorporating software adoption, the choice of firm scope, and

[3]Among others, see Aghion et al. (2023); Argente et al. (2021); Atalay et al. (2014); Bhandari et al. (2022); Bhandari and McGrattan (2020); Chiavari and Goraya (2023); De Ridder (2024); Ding et al. (2022); Farboodi et al. (2019); Weiss (2020).

[4]The non-homotheticity arises from the use of the CES production function and the treatment of software as a non-rival input. A recent literature considers non-homothetic production functions in understanding the rise in concentration, geographic divergence, and the welfare effects of trade, among others (Eckert et al., 2022; Lashkari et al., 2024; Trottner, 2020).

[5]Extensive research has delved into examining how ICT affects firm behavior and implications for the macroeconomy (e.g., Acemoglu et al., 2022; Aral et al., 2006; Aum and Shin, 2022; Baslandze, 2016; Bessen, 2020; Bloom et al., 2012; Brynjolfsson and Hitt, 1996, 2003; Brynjolfsson et al., 2023; Brynjolfsson and McElheran, 2016; Brynjolfsson and Yang, 1996; Contractor and Taska, 2023; Dedrick et al., 2003; Goldfarb and Tucker, 2019; Jorgenson, 2001; Jorgenson et al., 2003; Oliner and Sichel, 2000; Stiroh, 2002).

the non-rivalry of custom software. We show that the non-rivalry can provide a micro-foundation for the main mechanisms in both models: the assumption of a non-homothetic production function in Lashkari et al. (2024) and the assumption that intangibles lower the firm's marginal cost in De Ridder (2024). Moreover, examining the link with firm scope enables us to address the expansion of multi-establishment firms in the U.S., a key driver of the increase in concentration (Hsieh and Rossi-Hansberg, 2023; Smith and Ocampo, 2020). We find that the reallocation between adopters and non-adopters plays a crucial role in explaining the changes in concentration and that the software shock can account for 20 percent of the observed increase in the share of establishments owned by top firms.

The rest of the paper is organized as follows. Section 2 describes our data and sample. Section 3 presents the motivating facts on the relationship between the use of software and firm scope. Section 4 lays out the model, followed by model quantification in Section 5. Section 6 examines the implications of the software shock. Section 7 concludes.

2 Data

The data for this paper come primarily from two data sets: the Annual Capital Expenditures Survey (ACES) and the Revenue-Enhanced Longitudinal Business Database (LBD).

Annual Capital Expenditures Survey. The ACES is an annual firm-level survey available between 2002 and 2018 conducted by the Census Bureau that collects information on firms' capitalized investment in structures, equipment, and software. The survey gathers information on all sectors of the economy. Firms with over 500 employees are automatically sampled into the survey. Smaller firms are stratified by industry and payroll and then randomly selected. To ensure that our sample is nationally representative, we apply the weights provided by the ACES.

Software, as part of equipment investment, is reported in three categories: prepackaged, vendor-customized, and own-account. Prepackaged software is purchased off-the-shelf, vendor-customized software is externally developed and tailored to the firm's needs, and own-account software is created by the firm's employees for internal use. We focus on the latter two types—vendor-customized and own-account software (referred to as "custom software" hereafter)—which most closely map to our notion of an input that is non-rival and non-excludable within the firm, but excludable across firms. The ACES specifies that the firm should only report software developed for "internal use"—software that is developed to meet the firm's own needs—and should exclude investments in software that they plan to sell to the market. It also specifies that firms should only include capitalized investments, i.e., those listed as assets on the firm's balance sheet and then depreciated or amortized (U.S. Census Bureau, 2022).

We exclude pre-packaged software for two reasons. First, though prepackaged software is still non-rival, it is often excludable by the vendor. Often a firm has to buy a separate license for each person or establishment using the product. As a result, it does not scale with the scope of the firm like the input in our model that is non-rival and non-excludable within the firm. Second, investments in pre-packaged software are likely underreported in our data. This is due to the accounting guidelines for handling pre-packaged versus customized software. While vendor customized and in-house developed software should be capitalized on the balance sheet and therefore captured by the ACES, there are exceptions for pre-packaged software, which is often expensed. We discuss the accounting guidelines in Section A.2. In the model, we treat pre-packed software as a part of traditional capital.

One may be concerned that firms are not properly tracking and reporting their software investments or that firms are not capitalizing them onto their balance sheet. However, according to the Generally Accepted Accounting Principles (GAAP), both vendor-customized and internally developed software for internal use should be capitalized onto the firm's balance sheet.[6] We describe the ACES and the accounting principles in Appendix A. Additionally, in Appendix B.4, we restrict to a sample of public firms, which must follow GAAP guidelines in their financial statements to the Securities and Exchange Commission (SEC) and are less likely to be subject to measurement issues.

Longitudinal Business Database. We merge the ACES with the revenue-enhanced LBD, a panel dataset of the universe of U.S. employer establishments. The LBD contains information on establishment employment, payroll, age, industry, and location. Importantly, the LBD contains firm identifiers for each establishment, which allows us to aggregate to the firm level. When a firm is in more than one industry, we impute the industry of the firm using the one with the largest employment share.[7] The data also report sales at the firm level.

We use the number of establishments of the firm as the primary measure of firm scope. In Appendix B.2, we show that our results are robust to alternative measures of firm scope, such as employment, sales, and number of industries. Using the firm identifiers, we follow the procedure described in Rubinton (2020) to merge the LBD with the ACES.

[6]Prepackaged software, however, may be expensed. Thus, the ACES likely under-counts prepackaged software investments, which are often not capitalized. This is another reason why we focus on custom rather than pre-packaged software.

[7]Following Rubinton (2020), we assign the 2-digit NAICS code with the highest employment share first, and then assign the three-digit code with the highest employment share that is consistent with the 2-digit code. We then follow the same procedure up to 6-digits.

Sample selection. After merging the LBD and the ACES, we drop a number of observations, including those with: (1) zero or missing payroll, sales, or employment; (2) missing values of total capital expenditures, missing equipment or structures investment, or missing own-account, custom, or pre-packaged software investment; (3) firms that report negative fixed assets at the end of the year; (4) firms that report equipment investment that is less than software investment (software should be included in equipment). We also winsorize outliers of total capital expenditures, equipment, structures, each type of software investment, and custom software per employee at the 99.5th percentile in each year and 6-digit NAICS industry. Our final sample includes 384,000 observations.[8] Table A.1 displays the proportion of investment in the publicly released ACES totals that are accounted for by the firms included in our final ACES-LBD matched sample. Averaging across the years, our sample accounts for 71% of total software investment and 66% of total capital expenditures. Figure A.2 shows a strong correlation across sectors between software investments in our ACES sample and the BEA.[9]

Summary statistics. Table 1 reports summary statistics of software investment and other firm characteristics. We call firms that report positive investments in custom software adopters and firms with zero investment non-adopters. Approximately 3% of firms have adopted custom software.[10] These adopters devote a significant share of their total investment to custom software—on average, 37.8% of their total capital expenditure is devoted to custom software rather than traditional equipment and structures. The lower panel shows that software adopters are, on average, bigger; they have higher employment and sales and operate more establishments. Notably, the average number of establishments is six times larger for adopters than non-adopters.

Custom software investment across sectors. Figure 2 presents custom software expenditures across sectors. Panel (A) shows total expenditures by sector in 2017, with the Information, Finance, Professional and Scientific, Management, and Manufacturing sectors reporting the highest totals. Panel (B) tracks expenditure per employee for these five sectors every five years from 2002 to 2017.[11] Over time, these sectors exhibit a similar upward trend in software spending per employee, which rises by roughly threefold over the period.

[8] Observation counts are rounded to the nearest thousand in accordance with Census's disclosure review policies.

[9] The BEA estimates investment in prepackaged and vendor-customized software using a supply-side approach, based on software producers' sales and input-output tables. For own-account software, it relies on data from the BLS on employment in software development (e.g., programmers) and associated wages to impute capital formation.

[10] While this adoption rate might seem low, it aligns closely with existing literature. Bessen and Wang (2024) also document a similarly low adoption rate of custom software. Additionally, Acemoglu et al. (2022) report that in 2017, only about 2–3% of firms adopted AI or robots, while only 40% used specialized software, which includes widely used pre-packaged software like Square or QuickBooks. Finally, we note that custom software is 8% of non-residential investment, but highly concentrated. Together, these findings are consistent with our observed 3% adoption rate.

[11] We report the expenditure per employee for Census years, i.e., years ending in 2 and 7.

Table 1: Summary Statistics

	All Firms	Adopters	Non-adopters
Software Investment			
$\mathbb{1}$[adopting custom software]	0.030	1	0
	(0.170)	(–)	(–)
Custom SW to total capital expenditure share	0.027	0.378	0
	(0.135)	(0.352)	(–)
Custom SW expenditure per employee (thous. $)	0.050	1.690	0
	(0.910)	(5.00)	(–)
Other characteristics			
Total capital expenditure (million $)	0.265	5.497	0.104
	(27.0)	(145.2)	(10.0)
Equipment expenditure (million $)	0.166	3.557	0.062
	(22.1)	(121.4)	(7.1)
Structure expenditure (million $)	0.097	1.900	0.042
	(10.4)	(51.5)	(5.4)
Payroll (million $)	1.487	18.61	0.961
	(66.9)	(280.1)	(46.8)
Sales (million $)	7.423	102.90	4.485
	(475.6)	(2171)	(296.3)
Employment	30.7	334.8	21.3
	(1347)	(5989)	(873.2)
Number of establishments	1.473	7.747	1.279
	(28.09)	(132.8)	(16.42)

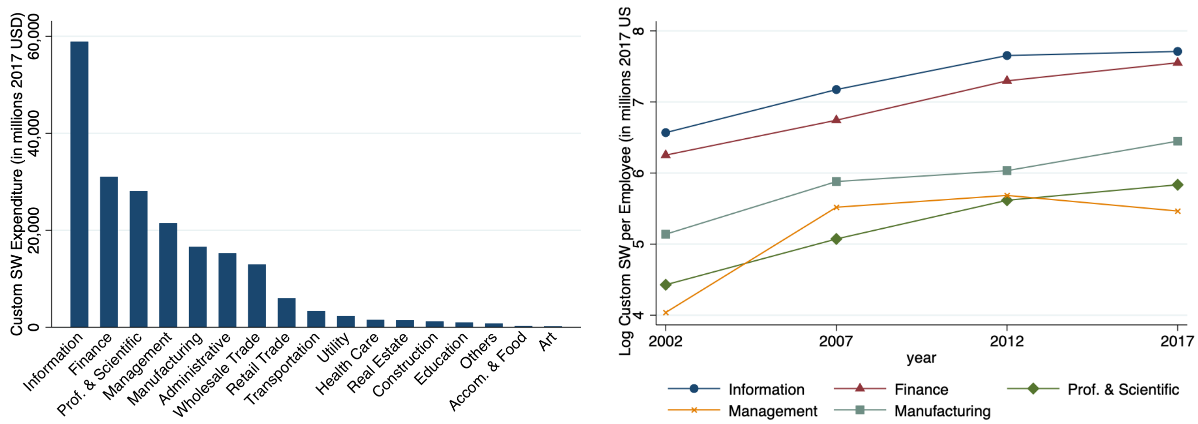
Notes: This table shows summary statistics of the LBD-ACES matched sample for all firms, custom software adopters, and non-adopters, respectively. Adopters are firms with positive investments in custom software, and non-adopters are firms with zero investment in custom software. Standard deviations are in parenthesis.

Despite this common growth pattern, there remains substantial sectorial heterogeneity in custom software investment. Figure 3 links this heterogeneity to market concentration by plotting software investment per employee against concentration across sectors. Panels (A) and (B) reveal a strong positive correlation: sectors that are more intensive in custom software—such as Information, Finance, and Utilities—also exhibit higher levels of sales and establishment concentration.

Panel (C) of Figure 3 presents regression estimates controlling for other forms of investment, including pre-packaged software, non-software equipment, and structures. The coefficient on custom software is large: a one log-point larger sector-level custom software investment is associated with an 8.3 percentage point larger share of establishments owned by the top 1% of firms. Columns (2) and (5) confirm that the relationship between custom software and concentration remains robust to controlling for pre-packaged software. Columns (3) and (6) add further controls for non-software equipment and structures, confirming that custom software, rather than other forms of investment, is most predictive of concentration across industries. Table A.2 shows that these patterns hold at the 3-digit NAICS industry level as well.

Figure 2: Custom Software Expenditures by Sectors

(A) Distribution of Custom Software Expenditures (B) Growth in Custom Software, Top 5 Sectors



Notes: Panel (A) shows the expenditures on custom software by sector in 2017. Panel (B) plots the log custom software expenditure per employee every five years from 2002 to 2017 for the Information, Finance, Professional and Science, Management, and Manufacturing sectors. Source: ACES, Longitudinal Business Database, and Economic Census.

3 Motivating Facts

In this section, we use the merged ACES-LBD data described in Section 2 to document new facts on the relationship between firm scope and custom software use. First, in Section 3.1, we show how software investment varies with firm scope on the extensive and intensive margins. Second, in Section 3.2, we show how the cost shares of software, capital, and labor vary with firm scope.

3.1 Software Intensity and Firm Scope

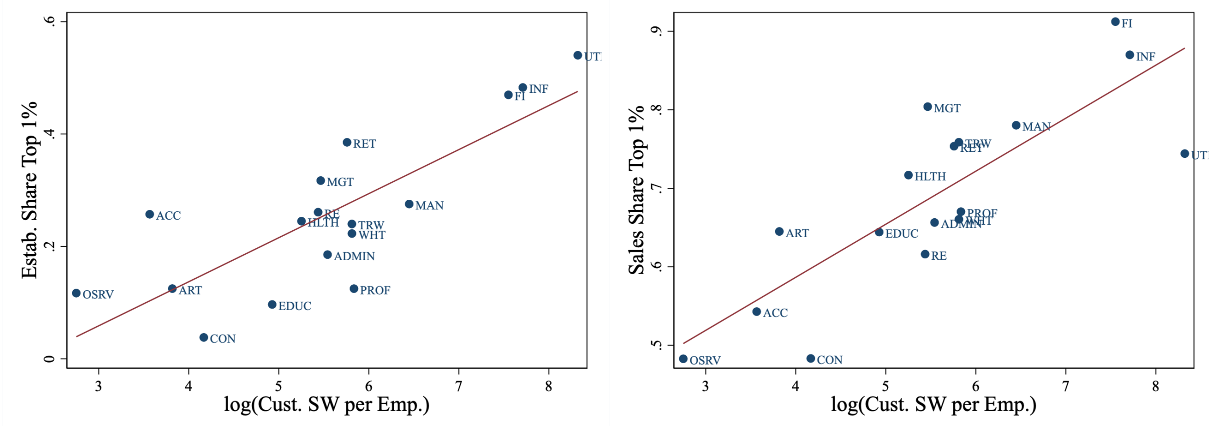
This section presents our main stylized facts. First, on the extensive margin, the likelihood that a firm has positive investments in custom software increases in firm scope, measured as the number of establishments a firm owns. Second, on the intensive margin, given adoption, the intensity of custom software investment, measured as the custom software share of total capital expenditures, decreases in firm scope.

We categorize firms into size bins based on the number of establishments they operate. Then, we estimate the following regression:

$$Y_{ikjt} = \gamma_k + \delta_{jt} + \varepsilon_{ikjt}, \tag{1}$$

Figure 3: Custom Software and Concentration Across Sectors

(A) Establishment Concentration (B) Sales Concentration



(C) Concentration and Investment

	Estab. Share Top 1%			Sales Share Top 1%		
	(1)	(2)	(3)	(4)	(5)	(6)
log(Custom SW)	0.0826***	0.102***	0.0948***	0.0928***	0.0957***	0.0971***
	(0.0142)	(0.0313)	(0.0234)	(0.0143)	(0.0269)	(0.0181)
log(Pre-Pack. SW)		-0.0248	-0.0467		-0.00366	-0.00801
		(0.0420)	(0.0337)		(0.0225)	(0.0205)
log(Equip., non SW)			0.0127			-0.0401
			(0.0350)			(0.0252)
log(Structures)			0.0349			0.0438**
			(0.0265)			(0.0167)
log(Employment)	-0.0625*	-0.0605*	-0.0475	-0.0587***	-0.0584**	-0.0422**
	(0.0320)	(0.0321)	(0.0279)	(0.0204)	(0.0206)	(0.0181)
Constant	0.651	0.642	0.183	0.940***	0.939***	0.693**
	(0.449)	(0.450)	(0.480)	(0.291)	(0.296)	(0.299)
Observations	70	70	70	70	70	70
R-squared	0.539	0.545	0.602	0.696	0.696	0.755
Year FE	Yes	Yes	Yes	Yes	Yes	Yes

Notes: Figures show the correlation between the share of establishments and sales allocated to the largest 1% of firms with custom software. Table shows regression version with further controls for pre-packaged software, non-software equipment, and structures. Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1. Source: ACES, Longitudinal Business Database, and Economic Census.

where i denotes the firm, k the establishment-size bin, j the industry, and t year. γ_k is a set of fixed effects for each establishment size bin, and δ_{jt} are industry-year fixed effects at the 6-digit NAICS level. Our main outcome variable, Y_{ijkt}, is either (1) an indicator for whether firm i has positive investments in custom software or (2) firm i's custom software intensity. In the first case, we include all firms, both adopters and non-adopters. In the latter case, we drop all firms that report zero custom software investment. In other words, we estimate the relationship between software intensity and firm scope on the intensive margin, conditional on the firm being an adopter.

Figure 4 shows our main results, plotting the establishment-size fixed effects, γ_k. Panel A shows that, on the extensive margin, the fraction of firms adopting custom software increases with the number of establishments of the firm. Compared to single-unit firms (the omitted category), firms with 2–4 establishments are 5.5 percentage points more likely to invest in custom software. The fraction further increases by over 40 percentage points for firms with over 100 establishments.

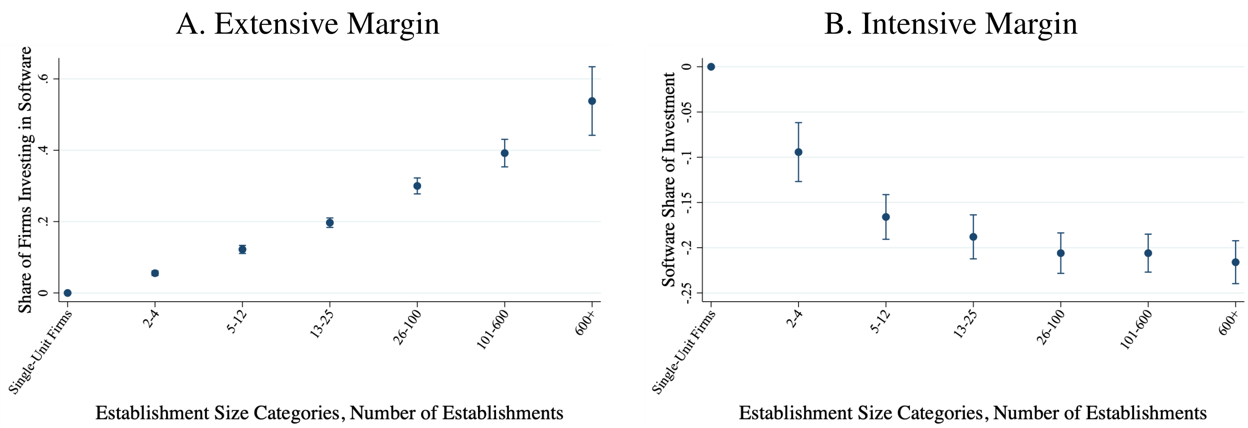
On the intensive margin, Panel B shows that conditional on positive investment, the intensity of custom software investment decreases with firm scope. Here, we measure the software intensity by the share of custom software expenditures relative to total capital expenditures. On average, adopters devote 37.8 percent of their capital expenditures to custom software. For firms with over 100 establishments, the custom software share of investment is 21 percentage points lower than single-unit firms. Despite companies with a greater number of establishments allocating a smaller *share* of their capital expenditure to custom software, the *level* of investment in custom software by these firms is higher.[12] For the largest firms, the investment share begins to stabilize, consistent with the model's predictions discussed in Section 5.2.

Robustness checks. We present four sets of robustness checks in Appendix B.1–B.4. First, on the intensive margin, Table A.3 shows that the results are robust to alternative measures of software intensity. Column (3)–(6) use the custom software expenditures per worker and the cost shares of custom software that account for software and capital stocks, as we explain in more detail in the next section. For the cost share, ideally we would observe the custom software stock, but the data only report investment flows. Moreover, many firms only show up once in the sample or have a short panel, which prevents us from using the perpetual inventory method.[13] Therefore, in column (4), our baseline measure proxies the software stock using its investment, assuming full depreciation within a year. Alternatively, assuming a depreciation rate less than one, column

[12]Lashkari et al. (2024) use firm-level microdata from France and document that the ICT cost share is increasing with firm size. Using U.S. data, we focus on a specific type of software investment—custom software—and find different empirical patterns, particularly on the intensive margin.

[13]While firms with more than 500 employees are intended to be sampled annually, their coverage in the data is uneven, limiting the usefulness of the panel. Restricting attention to firms observed for three to five consecutive years yields a sample with an average firm size that is more than 100 times the average firm in the LBD. As a result, we work directly with investment flows and consider alternative assumptions when constructing software cost shares.

Figure 4: Software Intensity and Firm Scope



Notes: This figure plots the establishment-size bin fixed effects, γ_k, from Equation (1)), with single-unit firms as the omitted category. Panel A plots the share of firms investing in custom software in each establishment size category. Panel B plots the software investment intensity, measured by the share of custom software expenditures relative to total capital expenditures, for each establishment size category. The regression controls for industry-year fixed effects at the 6-digit NAICS level. We report the 95% confidence interval. Standard errors are clustered at the industry-year level.

(5) computes the steady-state software stock by dividing the software investment by an industry-specific depreciation rate.

Additionally, since software expenditures, especially for own-account software, may take the form of wages to developers, one might be concerned about double-counting in total costs. To address this, in column (6), we construct an adjusted cost share that subtracts software investment from the wage bill in computing the cost share. Appendix B.1 provides further details on the construction of these measures. Across all measures, software intensity declines in firm scope.

Second, we show that the results are robust to alternative measures of firm scope, including firm employment, sales, and the number of industries in which the firm operates. As shown in Table A.4, the positive relationship on the extensive margin and the negative relationship on the intensive margin remain across different measures of firm scope.

Third, Table A.5 shows that our results are robust to incorporating firm fixed effects to account for time-invariant firm heterogeneity. Specifically, we restrict to a sample that includes firms with more than 500 employees and show up at least twice in the sample period. In this case, the coefficients are identified by within-firm variations in the software investment over time.

Finally, to address the concern that firms might not accurately track and capitalize custom software investments, we restrict our analysis to a sub-sample of public firms. Public firms, generally larger in scale, are obligated to follow GAAP guidelines in their financial statements to the SEC, alleviating potential concerns about measurement errors.[14] Table A.6 shows that the empirical patterns remain for public firms.

3.2 Cost Shares and Firm Scope

In addition to software, the cost shares of labor and capital also vary with firm scope. Let j denote the industry of firm i. We compute the firm's cost share of software by

$$\mu_{it}^{s} = \frac{r_{jt}^{s} s_{it}}{r_{jt}^{k} K_{it} + r_{jt}^{s} s_{it} + wL_{it}}, \tag{2}$$

where r_{jt}^{s} is the rental rate for custom software and r_{jt}^{k} is the rental rate for all other capital.[15] For the software input s_{it}, we use the firm's investment as the baseline measure for the stock, assuming software fully depreciates each year. In Appendix B.1, we show that our results remain robust under different assumptions regarding the depreciation rate of custom software and whether the firm's software expenditures take the form of wages to its developers or payments to a third-party vendor.

We use the firm's fixed assets reported in the ACES to measure the capital input K_{it}. Since software is included in the measures of equipment, we subtract software investment to get a measure of non-software capital. We use the payroll reported in the LBD to measure wage bills wL_{it}. The cost shares of labor and capital are similar, with wL_{it} or $r_{jt}^{k} K_{it}$ in the numerator of Equation (2).

Using our measures of the cost shares, we estimate regressions of the cost share on the log number of establishments:

$$\mu_{it}^{f} = \beta_1^{f} \mathbb{1}[\text{SW adopter}_{it}] + \beta_2^{f} \log(N_{it}) + \beta_3^{f} \mathbb{1}[\text{SW adopter}_{it}] \times \log(N_{it}) + \varepsilon_{it}, \tag{3}$$

where f refers to the specific factor (software, capital, or labor) and $\mathbb{1}[\text{SW adopter}]$ is an indicator equal to 1 if the firm is an adopter. Additionally, we control for firm age and industry-year fixed effects.

Table 2 Column (1) reports the results for the cost share of custom software. Since, by construction, this cost share is zero for non-adopters, we focus on the sample of adopters. For adopters, the coefficient on the log number of establishments is estimated at -0.004, which confirms the negative relationship between software intensity and firm scope. Since firms in the largest establishment-size bin are about 600 establishments (or 6.4 log points) bigger than single-unit

[14] We discuss the GAAP accounting standards in Appendix A.2.
[15] We use the rental rates at the 4-digit NAICS level from the BLS.

Table 2: Cost Shares and Firm Scope

	Cost Share of		
	Custom Software (1)	Labor (2)	Capital (3)
$\mathbb{1}$[SW adopter]		-0.019^{***}	-0.029^{***}
		(0.004)	(0.005)
$\mathbb{1}$[SW non-adopter] $\times \log(N_{\text{Estab}})$		0.006^{***}	-0.007^{***}
		(0.001)	(0.001)
$\mathbb{1}$[SW adopter] $\times \log(N_{\text{Estab}})$	-0.004^{***}	-0.007^{***}	0.018^{***}
	(0.001)	(0.002)	(0.002)
N	82,000	384,000	384,000
R^2	0.734	0.257	0.257
Ind–Year FE	Y	Y	Y
Age FE	Y	Y	Y

Notes: This table estimates Equation (3) with the cost share of custom software, labor, and capital as dependent variables. $\mathbb{1}$[SW adopter] is an indicator set to 1 if a firm makes positive investment in custom software, and $\log(N_{\text{Estab}})$ is the logarithm of the number of establishments the firm operates. We control for firm age and industry-year fixed effects. Industry is at the 6-digit NAICS level. Standard errors are clustered at the industry-year level. *** p<0.01, ** p<0.05, * p<0.1.

firms, their software cost share is, on average, about $2.6 (= 6.4 * 0.004)$ percentage points lower. The average custom software cost share for adopters is 4.5%. Thus, the software cost share is more than 50% lower for the largest adopters than the smallest adopters.

Columns (2)–(3) report the results for the cost shares of labor and capital. To ease interpretation, we report the coefficient on $\log(N_{it})$ for non-adopters (β_2) and adopters ($\beta_2 + \beta_3$), respectively. Interestingly, the labor cost share for non-adopters and adopters displays different relationships with firm scope. The estimated coefficient for non-adopters is 0.006, indicating that their labor cost share increases in the number of establishments. On the other hand, the coefficient for adopters is estimated to be negative at -0.007: The more establishments a firm operates, the lower its cost share of labor. A firm with 6.4 log-points more establishments has, on average, a labor cost share that is $4.5 (= 6.4 * 0.007)$ percentage points lower.

If both the labor and software cost shares are decreasing in firm scope for adopting firms, then the capital cost share must be increasing. This is confirmed in column (3). The capital share is 1.8 percentage points higher for firms with a 1 log point larger number of establishments.

These facts would be inconsistent with models that treat ICT as a fixed cost that increases the firms Hicks-neutral productivity. In this type of model, the labor and capital shares move together, and they cannot generate a capital share that rises with firm scope while the labor share falls. In the next section, we develop a model in which software is a non-rival variable input rather than a fixed cost, yielding a non-homothetic production technology that matches these empirical patterns.

4 Model

In this section, we present a theory of software as an input that is non-rival across the firm's establishments.[16] In Section 4.4, we discuss how the model can match the cross-sectional facts presented in Section 3.

4.1 Final Good Producer

A representative firm produces the final good in a perfectly competitive market by aggregating output y_i from a continuum of intermediate input producers i

$$Y = \left(\int_{\mathscr{I}} y_i^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{4}$$

where ε is the elasticity of substitution *across* firms $i \in \mathscr{I}$ and $\mathscr{I}$ is the set of producing firms that is endogenously determined in equilibrium. Each firm's output y_i is in turn a CES aggregator of the differentiated varieties y_{ie} produced by a continuum of its establishments $e \in [0, N_i]$

$$y_i = \left(\int_0^{N_i} y_{ie}^{\frac{\theta-1}{\theta}} de \right)^{\frac{\theta}{\theta-1}}, \tag{5}$$

where θ is the elasticity of substitution *within* the firm across establishments. N_i is the measure of establishments that firm i chooses to operate, i.e., firm scope.

The final good producer purchases intermediate goods produced by establishments at price p_{ie}. Profit maximization implies that the demand facing each establishment is

$$y_{ie} = \left(\frac{p_{ie}}{p_i} \right)^{-\theta} \left(\frac{p_i}{P} \right)^{-\varepsilon} Y, \tag{6}$$

where the price indices are given by

$$P = \left(\int_{\mathscr{I}} p_i^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}} \quad \text{and} \quad p_i = \left(\int_0^{N_i} p_{ie}^{1-\theta} de \right)^{\frac{1}{1-\theta}}. \tag{7}$$

We use the final good as the numeraire and normalize its price, P, to 1.

[16] We summarize the model environment in Table A.8.

4.2 Intermediate Good Producers

Firm i operates a continuum of establishments $e \in [0, N_i]$, with each establishment producing a differentiated variety. Firms compete monopolistically in each of those markets. A firm can choose between two available technologies for their production function. Firms that do not adopt the new technology produce using a CES production function over capital and labor, whereas adopters also use custom software. We assume that the firm's establishments are identical.

Firms differ in their fundamental productivity z_i. Given productivity, firms maximize their profits by choosing whether to adopt custom software ($\tau_i \in \{NA, A\}$), their firm scope (N_i), the price and quantity of each establishment's variety, (p_{ie} and y_{ie}), and factor inputs at each establishment, including capital (k_{ie}), labor (l_{ie}), and if they are adopters, software (s_{ie}). Though all the choices are made jointly, we can solve the firm's problem backwards. First, conditional on the choice of scope, price, and technology, we solve the firm's cost minimization problem. Second, conditional on the technology choice, we solve for the choice of scope and price. Finally, we solve the technology adoption decision.

Production function. If a firm chooses not to adopt software (i.e., "non-adopters"), its establishments produce output using labor and capital

$$y_{ie}^{NA} = z_i \left[\gamma_l^{\frac{1}{\sigma_l}} l_{ie}^{\frac{\sigma_l - 1}{\sigma_l}} + (1 - \gamma_l)^{\frac{1}{\sigma_l}} k_{ie}^{\frac{\sigma_l - 1}{\sigma_l}} \right]^{\frac{\sigma_l}{\sigma_l - 1}}, \ \forall e \in [0, N_i], \tag{8}$$

where the elasticity of substitution between capital and labor is given by σ_l and the weight on labor, as opposed to capital, is given by γ_l.

If a firm chooses to adopt software (i.e., "adopters"), its establishments produce output using labor, capital, and software

$$y_{ie}^{A} = z_i \left[\gamma_l^{\frac{1}{\sigma_l}} l_{ie}^{\frac{\sigma_l - 1}{\sigma_l}} + (1 - \gamma_l)^{\frac{1}{\sigma_l}} \left(\gamma_k^{\frac{1}{\sigma_k}} k_{ie}^{\frac{\sigma_k - 1}{\sigma_k}} + (1 - \gamma_k)^{\frac{1}{\sigma_k}} s_{ie}^{\frac{\sigma_k - 1}{\sigma_k}} \right)^{\frac{\sigma_k}{\sigma_k - 1} \frac{\sigma_l - 1}{\sigma_l}} \right]^{\frac{\sigma_l}{\sigma_l - 1}}, \ \forall e \in [0, N_i]. \tag{9}$$

Unlike non-adopters, adopters now have an inner CES-bundle over capital and software, where the elasticity of substitution is σ_k and the weight on capital, as opposed to software, is γ_k. The outer bundle, which is CES over the labor and the capital-software bundle, is the same as for non-adopters.

Unit cost of production. Taking the wage w, the rental rate of capital r^k, and the rental rate of software r^s as given, a non-adopter chooses the labor and capital inputs at each of its establishments to minimize the total cost of production

$$\min_{k_{ie},l_{ie}} r^k N_i k_{ie} + w N_i l_{ie}, \tag{10}$$

subject to (8). The cost minimization problem for adopters is

$$\min_{k_{ie},l_{ie},s_{ie}} r^k N_i k_{ie} + w N_i l_{ie} + r^s s_{ie}, \tag{11}$$

subject to (9). The key difference between adopters and non-adopters is in the treatment of custom software in the total cost. As labor and capital are rival inputs, a non-adopter firm must purchase $N_i k_{ie}$ units of capital in order to use k_{ie} units of capital at each establishment; in the firm's cost minimization problem, each of the rival inputs is multiplied by the firm's scope, N_i. On the other hand, because custom software is non-rival, its cost for adopter firms does not increase with the number of establishments. The firm only needs to spend $r^s s_{ie}$ in order to use s_{ie} units of software at each establishment. In Section 6.4, we extend the model to allow for specificity and partial excludability, relaxing the assumption that the firm can use the same software simultaneously and costlessly across all of its establishments.

Solving the minimization problem, the unit cost to the firm is

$$C_i^\tau(z_i, N_i) = \frac{1}{z_i}\left[(1-\gamma_l)(p_X^\tau(N_i))^{1-\sigma_l} + \gamma_l w^{1-\sigma_l}\right]^{\frac{1}{1-\sigma_l}}, \tau \in \{NA, A\} \tag{12}$$

where $p_X^\tau(N_i)$, given by,

$$p_X^{NA} = r^k \quad \text{and} \quad p_X^A(N_i) = \left[\gamma_k (r^k)^{1-\sigma_k} + (1-\gamma_k)\left(\frac{r^s}{N_i}\right)^{1-\sigma_k}\right]^{\frac{1}{1-\sigma_k}} \tag{13}$$

is the rental rate of capital for non-adopters and the unit cost of the inner bundle of capital and software for adopters.[17] It is noteworthy that the unit cost of production at the establishment level is the same as that at the firm level.[18]

Figure 5 plots the unit cost for non-adopters (black solid line) and adopters (green dashed line), respectively, against the firm's number of establishments for a firm with average productivity, $\bar{z}$.

[17] Appendix C.1 provides detailed derivation of the firm's problem.

[18] Denote the unit cost of production at the establishment by C_{ie}. Then, we can express the unit cost of production at the firm as total cost divided by total output, $C_i = C_{ie} y_{ie} N_i / (y_{ie} N_i)$, which is the same as C_{ie}.

Notably, the unit cost is constant for non-adopters regardless of their scope. On the other hand, the unit cost is decreasing in N_i for adopters. Thus, as the firm's scope increases, it will eventually become cost-effective for the firm to adopt custom software.[19]

Marginal cost of production. For adopters, the firm's marginal cost of production is

$$MC_i^A = \frac{\partial C_i(z_i, N_i(y_i))y_i}{\partial y_i} = C_i(z_i, N_i(y_i)) + \frac{\partial C_i(z_i, N_i(y_i))}{\partial y_i} y_i. \tag{14}$$

Here, we express firm scope as an implicit function of firm output, i.e., $N_i(y_i)$. The second term $\frac{\partial C_i(z_i, N_i(y_i))}{\partial y_i} < 0$ because the unit cost of production $C_i(z_i, N_i(y_i))$ decreases in firm scope N_i and thus output y_i. This leads to increasing returns to scope due to the fact that software is non-rival across the firm's establishments.

Conditional on firm scope N_i, an establishment's marginal cost of production is constant and equals the unit cost of production in Equation (12).

Cost share of software. The cost share of software is given by

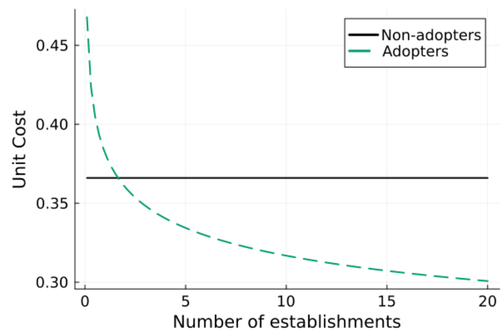
$$\mu_i^{\tau} = \begin{cases} 0, & \tau_i = NA \\ \frac{(1-\gamma_l)(p_X^A(N_i))^{1-\sigma_l}}{(1-\gamma_l)(p_X^A(N_i))^{1-\sigma_l} + \gamma_l w^{1-\sigma_l}} \times \frac{(1-\gamma_k)(\frac{r^s}{N_i})^{1-\sigma_k}}{(p_X^A(N_i))^{1-\sigma_k}}. & \tau_i = A \end{cases} \tag{15}$$

For adopting firms, the first term represents the cost share of the capital–software bundle in total costs, while the second term captures the share of software within that bundle. Importantly, the software cost share varies with the firm's scope N_i, implying that the production function is non-homothetic. In contrast, under a homothetic production function—such as a standard CES specification without non-rival inputs—cost shares remain constant. In our setting, software is non-rival, which introduces non-homotheticity into the production function. Whether the cost share is increasing or decreasing in firm scope will depend on the elasticities of substitution between factors, σ_k and σ_l. We will further discuss the relationship between the factor shares and firm scope in Section 4.4.

Firm scope. Given the unit cost for adopters and non-adopters, we now solve for the firm's choice of scope and price. We assume firms need to pay a fixed cost of production F^C to keep operating. In addition, maintaining multiple establishments incurs a span-of-control cost $F^N(N_i)$. The firm

[19] As the number of establishments grows to infinity, the effective rental rate of software goes to zero, and the unit cost of the inner capital–software bundle, $p_X^A(N_i)$, goes to $\gamma_k^{1/(1-\sigma_k)} r^k$.

Figure 5: Unit Cost Versus Firm Scope



Notes: The figure shows the unit cost of production for non-adopters and adopters versus firm scope for a representative firm with average productivity, $\bar{z}$.

chooses its scope N_i and its price at each establishment p_{ie} to maximize profits. It solves

$$\Pi^\tau(z_i) = \max_{N_i, p_{ie}} N_i p_{ie} y_{ie} - N_i C_i^\tau(z_i, N_i) y_{ie} - F^N(N_i) - F^C, \tag{16}$$

subject to its downward-sloping demand curve for the variety produced by each establishment, given by Equation (6).[20]

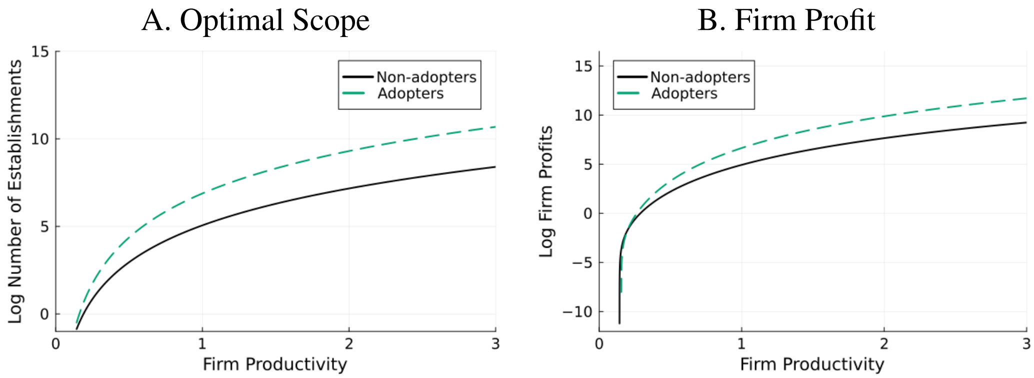
While there is no closed-form solution for the choice of N_i, the first-order condition with respect to N_i can be written as

$$\underbrace{\pi_{ie}^\tau}_{\text{profits per estab.}} \left[1 - \underbrace{\frac{\theta-\varepsilon}{\theta-1}}_{\text{within-firm cannibalization}} + \underbrace{(\varepsilon-1)\mu_i^\tau}_{\text{cost reduction}}\right] = \underbrace{\frac{\partial F^N(N_i)}{\partial N_i}}_{\text{span of control cost}}, \tag{17}$$

where π_{ie}^τ is the profits per establishment given by $\pi_{ie}^\tau = \frac{1}{\varepsilon}\left(\frac{\varepsilon}{\varepsilon-1}\right)^{-\varepsilon} P^\varepsilon Q (N_i^\tau)^{\frac{\theta-\varepsilon}{1-\theta}} (C_i^\tau(z_i, N_i))^{1-\varepsilon}$. The marginal benefit of adding an additional establishment (the left-hand side) is given by the additional profits of the establishment, π_{ie}^τ, which is then augmented by the term in brackets. The second term, $\frac{\theta-\varepsilon}{\theta-1}$, captures the decreasing returns to an additional establishment when $\theta > \varepsilon > 1$, arising from the within-firm cannibalization effect. The third term $(\varepsilon-1)\mu_i^\tau$ captures the increasing returns-to-scope from the non-rivalry of custom software. μ_i^τ is the firm's software cost share in Equation (15). Because μ_i^τ is positive for adopters, it increases the marginal benefit of an additional establishment and the optimal span of control for adopters will be larger. However, in order to match the data, μ_i^τ will be decreasing in the scope of the firm, so this additional benefit

[20] As we assume all establishments of the firm are identical, the demand facing each establishment becomes $y_{ie} = N_{ie}^{\frac{\theta-\varepsilon}{1-\theta}} p_{ie}^{-\varepsilon} P^\varepsilon Q$, where P and Q are the price index and aggregate demand, respectively.

Figure 6: Firm Scope and Profit



Notes: This figure plots the logarithm of optimal firm scope N_i (Panel A) and firm profits (Panel B) against firm productivity z, for adopters and non-adopters, respectively.

dissipates as the firm grows. The μ_i^τ is multiplied by ε, the elasticity of substitution between firms because when the elasticity is larger, the demand for the firm's varieties increases more in response to a cost reduction. The right-hand side is the marginal span of control cost.

Panel A of Figure 6 plots the log of optimal firm scope for non-adopters and adopters, respectively, against firm productivity. More productive firms choose a larger scope, conditional on being adopters or non-adopters. However, conditional on the same productivity, adopters choose a larger scope than non-adopters because of the cost reduction from the non-rival custom software input.

Pricing rule. Each establishment of the firm faces a CES demand curve given by Equation (6) and engages in monopolistic competition. The firm chooses its price at each establishment to maximize profits, according to Equation (16). Then, the optimal price is a constant markup over the marginal cost of production at the establishment, which is the same as the unit cost in Equation (12):

$$p_{ie}^\tau = \frac{\varepsilon}{\varepsilon - 1} C_i^\tau(z_i, N_i), \qquad \tau \in \{NA, A\}. \tag{18}$$

Adoption of custom software. Finally, the firm chooses whether to adopt custom software by comparing profits under adoption and non-adoption. Panel B of Figure 6 plots the log of firm net profits against firm productivity for non-adopters and adopters. Notably, the model delivers an extensive-margin adoption decision driven by increasing returns to scope from the non-rival nature of custom software. When firm productivity is low, the optimal scope of the firm is low. As a result, the unit cost of using the non-rival custom software technology is high, as shown in Figure 5. As firm productivity increases, the optimal scope of the firm increases. The profit associated with adopting becomes higher as the cost of the software is shared across many establishments. Eventually, the decrease in the unit cost from the non-rival input is sufficient to make adoption worthwhile.

While not essential for generating the adoption decision, we introduce a fixed adoption cost F_i^S to match the share of adopters in the data. We assume this fixed cost is a random variable drawn from distribution, $G^s(F)$, and is independently and identically distributed across firms. This assumption helps match the observed extensive margin fact: even among firms with similar scope, some choose to adopt custom software while others do not.

The firm chooses to adopt custom software if the profit associated with adopting, net of the fixed cost, is higher than not adopting:

$$\Pi(z_i) = \max\{\Pi^{NA}(z_i), \Pi^{A}(z_i) - F_i^S\}, \tag{19}$$

where the gross profits are given in Equation (16).

Entry and exit. Following Melitz (2003), we add entry and exit of firms to the model, assuming that there is an unbounded mass of potential entrants and that firms draw an idiosyncratic productivity from a distribution, $g(z)$, after incurring a sunk entry cost F^E. If their productivity is too low, it will not be worth paying the fixed cost of production, and they will immediately exit. If they start to produce, they face a constant probability of an exit shock δ. Firms will exit when their productivity is below a threshold, z^*, given by

$$\frac{1}{\delta}\Pi(z^*) = 0. \tag{20}$$

The distribution of producing firms will be given by $\tilde{g}(z) = \frac{g(z)}{1-G(z^*)}$.

A free-entry condition must hold so that the marginal entrant is indifferent between entering and staying dormant,

$$\frac{1-G(z^*)}{\delta}\int_{z^*}^{\infty}\Pi(z)\tilde{g}(z)dz = F^E, \tag{21}$$

where the left-hand side gives the expected value of an entering firm before receiving its idiosyncratic productivity shock.

4.3 General Equilibrium

A representative household inelastically supplies one unit of labor to the intermediate good producers and consumes the final good. We assume that a representative firm transforms the final good into capital and software at rates Z_k and Z_s, respectively. The capital and software markets are perfectly competitive, so the rental rate for capital will be $r = P/Z_k$, and the rental rate for software will be $r^s = P/Z_s$, where P is the price of the final good.[21]

The final goods market clearing condition is given by

$$Y = C + \frac{1}{Z^k} K + \frac{1}{Z^s} S + F. \tag{22}$$

Consumption of the representative household, C, is equal to the wage, w, plus any profits from intermediate good producers and software and capital good producer that are remitted to the consumer.[22] K is the aggregate demand for capital given by $K = M \int_z k^d(z)\tilde{g}(z)dz$, where $k^d(z)$ is the optimal demand for capital for firms with productivity z. Similarly, S is the aggregate demand for software given by $S = M \int_z s^d(z)\tilde{g}(z)dz$. F is the aggregate costs paid by the firms defined as[23]

$$F = \underbrace{\frac{M\delta F^E}{1-G(z^*)}}_{\text{entry costs}} + \underbrace{M \int_z F^N(N_i(z))\tilde{g}(z)dz}_{\text{span-of-control costs}} + \underbrace{MF^C}_{\text{fixed costs of production}} + \underbrace{MF^S \int_z \mathbb{I}[\tau = A]\tilde{g}(z)dz}_{\text{fixed costs of adopting software}}. \tag{23}$$

In our baseline model, we assume that the fixed costs are denominated in units of output. In Appendix D.4, we show that the main results are robust to denominating them in units of labor.

Definition. A general equilibrium of the economy consists of the price of the final good, P; the wage, w; the rental rate of capital, r^k; the rental rate of software, r^s; the mass of firms, M; an exit threshold, z^*; and an adoption threshold, z^A, such that

- firms choose whether to exit, price, scope, technology choice, and factor shares according to (20), (18), (17), (10), (11), and (19);
- free entry (21) and zero profit conditions (20) hold;
- the capital, labor, software, final good, and intermediate goods markets clear.

4.4 Cost Share and Firm Scope

In this section, we show that the model can match the key stylized facts that we document in the empirical section. Namely, that the likelihood of adopting custom software is increasing in the scope of the firm and that, conditional on adopting, the cost share of software is decreasing in firm scope.

[21] The model environment is summarized in Appendix Table A.8.

[22] In the baseline model, the software and capital good producer does not make profits, but they will in the extension in Section 6.4.

[23] We assume the costs are paid in final goods. Alternatively, one can assume that these costs are paid in labor.

Extensive margin. In the model, firms with larger scope are more likely to adopt custom software. As shown in Figure 5, the unit cost falls as the scope increases for adopting firms because the software investment cost gets shared across multiple establishments. This leads to a productivity threshold at which the profit from adopting exceeds that of not adopting. Figure 6 Panel B illustrates that, for a given z, profits from adoption eventually surpass those from non-adoption, and moreover, the gap grows with productivity. This result holds even when there is no fixed cost to adopting the new technology; introducing a fixed cost simply shifts the adopters' profit curve (the green dashed line) up and down but does not change the shape.

In our baseline model with random fixed costs, no firms adopt software below the productivity threshold; above this threshold, a firms adopts software when the profit gain from adoption relative to non-adoption exceeds its fixed cost draw. As the profit gap widens with productivity, the probability that a firm adopts software also increases. This positive relationship between firm scope and adoption of the custom software matches the extensive margin pattern as in Panel A of Figure 4.

Intensive margin. For adopting firms, the cost shares of software, capital, and labor vary endogenously with firm scope N_i. The cost share of custom software relative to the cost share of capital is

$$\frac{r^s s_{ie}}{r^k N_i k_{ie}} = \left(\frac{r^k}{r^s/N_i}\right)^{\sigma_k - 1} \frac{1-\gamma_k}{\gamma_k}. \tag{24}$$

Importantly, the relationship between the cost share of software relative to capital and firm scope N_i depends on the elasticity of substitution between capital and software, σ_k. Panel A of Figure 7 demonstrates that the software share relative to capital share is decreasing in N_i if $\sigma_k < 1$ (complements) and is increasing in N_i if $\sigma_k > 1$ (substitutes). If $\sigma_k = 1$, the production function is Cobb-Douglas, and the cost shares are constant.

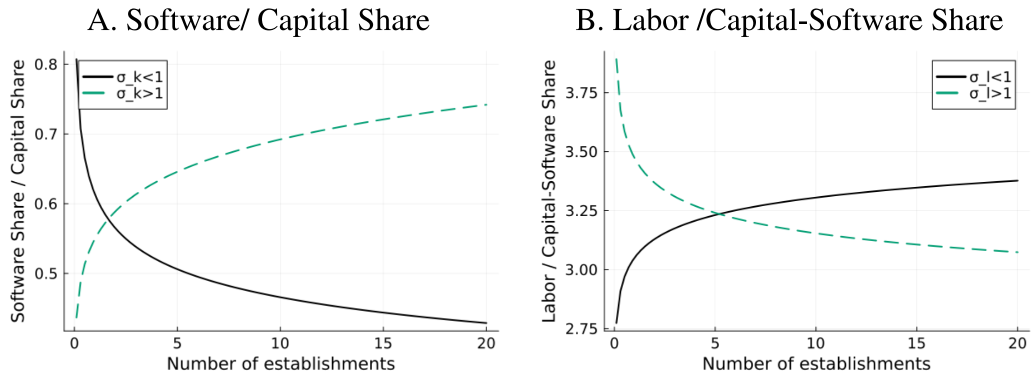
In Section 3.2, we document that conditional on adopting, the relative cost share of software to capital is falling with the firm's scope. The model will match this fact in the case where σ_k is less than 1. We use this insight in the next section to calibrate σ_k to match the negative correlation between software cost share and firm scope.

The same logic applies to the relationship between labor cost share and firm scope. The cost share of labor relative to the capital-software bundle is given by

$$\frac{w N_i l_{ie}}{r^k N_i k_{ie} + r^s s_{ie}} = \left(\frac{p_X^A(N_i)}{w}\right)^{\sigma_l - 1} \frac{\gamma_l}{1-\gamma_l}, \tag{25}$$

where $p_X^A(N_i)$ is given in Equation (13). This equation is plotted in Panel B of Figure 7 for the case when $\sigma_l > 1$ and the case when $\sigma_l < 1$. The labor share will decline with the scope of the firm

Figure 7: Software Intensity and Firm Scope



Notes: Panel A plots the cost share of custom software relative to capital against the firm's number of establishments. The black solid line corresponds to an elasticity of substitution between custom software and capital (σ_k) less than 1; The green dashed line corresponds to σ_k greater than 1. Panel B plots the cost share of labor relative to the sum of custom software and capital against the firm's number of establishments. The black solid line corresponds to an elasticity of labor and the capital bundle (σ_l) less than 1; The green dashed line corresponds to σ_l greater than 1.

relative to the capital-software share when $\sigma_l > 1$. In Section 3.2, we document that, conditional on adopting, the labor share is declining with firm scope. We use this insight to calibrate the value of σ_l in the next section.

Bundling in the CES-Production Function. In our baseline model, we assume that the establishment's production function follows Equation (9), a nested-CES in which labor is combined with a capital-software bundle. Alternatively, one could assume that capital is combined with a software-labor bundle.[24] In Appendix D.4, we recalibrate the model under this alternative bundling assumption and show that the aggregate implications are similar.

5 Quantifying the Model

In this section, we bring the model to the data. We first describe our parametrization and calibration strategy in Section 5.1. Then, in Section 5.2, we show the model implications for the cross-sectional relationships between firm scope and cost share of labor and the investment share of software.

[24] A third option would have capital and labor in the inner bundle and software in the outer bundle. However, this would not allow us to match our empirical fact that labor is decreasing in the scope of the firm while capital is increasing in the scope of the firm.

5.1 Parametrization

Our parametrization strategy proceeds in two steps. First, we make several assumptions on the functional forms and assign values to a set of parameters guided by the literature and data. Second, we internally calibrate the remaining parameters using the method of moments.

Firm productivity distribution. Following the convention in the literature, we assume that the firm productivity follows a Pareto distribution $g(z)$ with tail parameter α.

Span-of-control costs. We assume that the cost of managing establishments is log-linear in the number of establishments

$$F^N(N) = \omega_1 N^{\omega_2}, \text{ where } \omega_1 > 0, \omega_2 > 0. \tag{26}$$

Here, ω_1 governs the average span of control cost, and ω_2 captures the curvature with which the cost increases with N.

Fixed costs. We assume that the fixed cost of adopting custom software follows a log-normal distribution, $\log N(\bar{F}^S, \psi^2)$, where $\bar{F}^S$ and ψ^2 are the mean and variance of the log fixed cost distribution, respectively.

Assigned parameters. Panel A of Table 3 shows the assigned parameters. We set the elasticity of substitution across firms ε to 4, a standard value as in Head and Mayer (2014).[25] The exit probability δ is set to 1.9% to match the aggregate employment-weighted exit rate of firms in the Business Dynamic Statistics. The productivity of the custom software-producing and capital-producing sectors is set to match the rental rate of capital from the BLS.[26]

Calibration. We internally calibrate the remaining 12 parameters using the method of moments, summarized in Panel B of Table 3. We denote the vector of parameters $\Psi = \{\sigma_k, \sigma_l, \omega_1, \omega_2, \theta, F^E, F^C, \bar{F}^S, \gamma_k, \gamma_l, \alpha, \psi\}$, the data moments as the vector m, and the model moments as $\hat{m}(\Psi)$. Then, the calibrated $\hat{\Psi}$ minimizes the criterion function

$$f(\Psi) = [m - \hat{m}(\Psi)]' W [m - \hat{m}(\Psi)]. \tag{27}$$

We use the identity matrix as the weighting matrix W.

Identification. While all parameters are jointly determined, some moments are more informative for given parameters. Here, we provide a brief description of identification.

[25] Through the lens of the model, the markup is $\frac{\varepsilon}{\varepsilon-1}$. Then, $\varepsilon = 4$ implies a 33% markup, consistent with recent estimates of markups (e.g. De Loecker et al. (2020)).

[26] Appendix A.3 provides details on the construction of the rental rates for different types of capital.

Table 3: Parameterization

Panel A. Assigned Parameters

Parameter	Description	Source	Value
ε	Elasticity of sub. across firms	Head and Mayer (2014)	4
δ	Exogenous exit probability	Emp.-weighted firm exit rate, BDS	1.9%
Z_s	Productivity of custom SW sector	Rental rate of software, BLS	6.76
Z_k	Productivity of capital sector	Rental rate of capital, BLS	11.11

Panel B. Calibrated Parameters

Parameter	**Description**	**Value**	**Moment**	**Data**	**Model**
σ_k	Elasticity of sub. btw. k and s	0.881	Cross-section of sw share	-0.004	-0.004
σ_l	Elasticity of sub. btw. l and k-s	1.120	Cross-section of labor share	-0.007	-0.007
ω_1	Span of control cost	0.06	Establishments per firm	1.47	1.45
ω_2	Span of control curvature	1.29	Estab share top 1%	0.28	0.29
θ	Cannibalization	10.89	Sales share top 1%	0.63	0.63
γ_k	Weight on K	0.63	Inv share custom SW	0.10	0.10
γ_l	Weight on L	0.76	Labor share	0.56	0.56
F^E	Entry cost	20.20	Employees per firm	30.7	29.8
F^C	Fixed cost	0.07	Exit rate, age 1	0.21	0.21
$\bar{F}^S$	Location of log-normal for FC	32.82	Share adopting	0.03	0.03
ψ	Scale of log-normal for FC	33.48	Adoption 600+ rel. to 13–25	0.34	0.37
α	Pareto tail productivity	4.75	Pareto tail employment	1.10	1.11

Notes: This table summarizes model parameters. Appendix D.1 gives more detail on the source for each data moment.

(a) Elasticity of substitutions σ_k and σ_l: As discussed in Section 4.4, whether the software and labor cost shares decrease with firm scope depends on the magnitude of σ_k and σ_l. We calibrate the elasticities of substitution so that the model matches the cross-sectional relationships of the software- and labor-cost shares with firm scope shown in Section 3.2. We discuss how the calibrated values of these parameters compare to the literature in Appendix D.3.[27]

(b) Span-of-control costs ω_1 and ω_2: These two parameters jointly impact the distribution of establishments per firm. A higher ω_1 leads to a smaller number of establishments per firm, on average. A higher ω_2 particularly affects top firms with many establishments. We use the average number of establishments per firm and the share of establishments owned by the top 1% of firms to pin down these two parameters.

(c) Within-firm elasticity of substitution θ: We refer to θ as the cannibalization parameter, which

[27] Our choice of the nested CES structure over capital, software, and labor assumes that the elasticity between capital and labor is the same as that between software and labor. We show in Appendix D.4 that a calibrated model with alternative structure (i.e., labor is first combined with capital and then the labor-capital composite is combined with software) gives similar quantitative results.

controls the extent to which a firm can increase its sales and market share by expanding its number of establishments. We use the sales concentration, i.e., the sales share by the top 1% of firms, to calibrate this parameter.

(d) Weights on capital and labor γ_k and γ_l: We calibrate these two parameters to match the aggregate labor share and the aggregate share of non-residential fixed investment (excluding non-software intellectual property investments) that goes towards custom software.[28]

(e) Entry cost and fixed costs, F^E and F^C: The entry cost and the fixed cost of production jointly determine the average number of employees per firm and the exit rate of new entrants.

(f) Mean and variance of log fixed adoption costs, $\bar{F}^S$ and ψ: The mean is identified from the overall share of firms that invest in custom software, while the variance is pinned down by the extensive-margin relationship between adoption probability and firm scope—specifically, by comparing the adoption rates of firms with more than 600 establishments to those with 13-25. When there is no variance in fixed costs, only large firms adopt software. As variance increases, smaller-scope firms may draw lower fixed costs and find adoption worthwhile, narrowing the adoption gap between large and small firms.

(g) Pareto tail of the productivity distribution α: We calibrate α so that the Pareto tail for employment is within the range of estimates from Kondo et al. (2023) using Axtell's method.

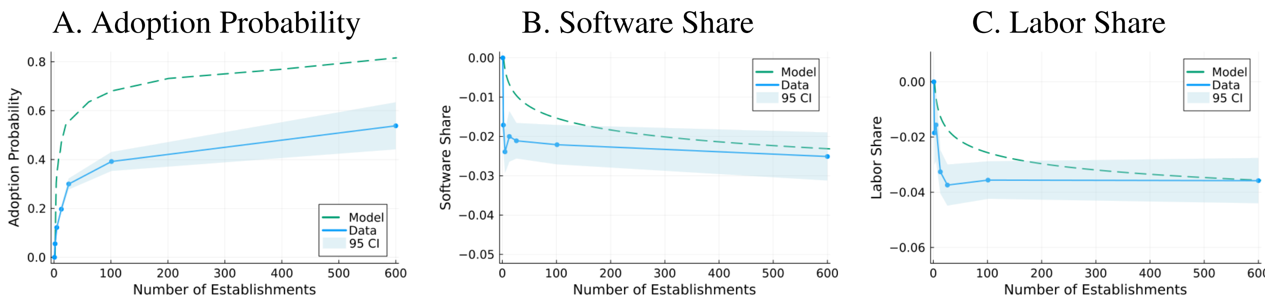
In addition, Appendix D.1 provides further information on the construction of the data moments, and Appendix Figure A.5 shows how each moment changes in response to a small increase in each of the parameters, holding all other parameters fixed.

Model fit. The last two columns of Panel B in Table 3 report the data moments and model-simulated moments, respectively. Overall, the model does a good job of matching these moments.

In addition to the targeted moments, our model is able to match the overall distribution of firm scope, which is not targeted in the calibration. Appendix figure A.4 shows the log average number of establishments for each employment size bin. While we target the share of establishments that are operated by the top 1% of firms, we do not target the full distribution of establishments per firm. Given that software is non-rival across a firm's establishments, it is important to match the establishment distribution in the data for the quantitative assessment of the importance of software. Overall, the model does a good job of matching the relationship between the number of establish-

[28] We exclude non-software intellectual property products because they are also forms of non-rival investments and, therefore, not accounted for in our model. Excluding those investments increases the investment share of software from approximately 8 to 10 percent.

Figure 8: Software Intensity and Factor Shares in the Model and Data



Notes: This figure plots the software adoption probability (Panel A), cost share of software (Panel B) and labor (Panel C), relative to those of the smallest firm, against the firm's number of establishments. The blue solid line corresponds to the regression estimates associated with software adopters in columns (1)–(2) of Table A.7, normalized by coefficients on the smallest establishment size bin. The green dashed line corresponds to the model simulation.

ments and firm size. However, the model slightly overestimates firm scope in the middle of the distribution, not quite generating the convexity of the data.

5.2 Software Intensity, Factor Shares and Firm Scope

The calibrated model can generate the empirical patterns we document in Section 3. Specifically, it can match the facts that the adoption probability increases in the number of establishments of the firm and that, for adopters, the cost shares of software and labor are both decreasing with the number of establishments of the firm. In the data, we observe that some single-establishment firms have adopted custom software. However, in the model, only firms with productivity above a certain threshold opt for adoption, and no single-establishment firms adopt software. The smallest adopter in the calibrated model has 2.3 establishments. To facilitate a meaningful comparison to the data, we present the adoption probability, software investment share and the labor cost share relative to the smallest adopter.

Panel A plots the probability of software adoption against the number of establishments of the firm. The blue dots represent the empirical counterparts of the software adoption probability relative to single-establishment adopters for each establishment size bin. For instance, the second dot from the left indicates that the adoption probability is 5.5 percentage points higher than single-establishment firms. Compared to the data, the model overshoots the rise in adoption probability for small firms with fewer than 12 establishments (the model line is too steep relative to the data for small firms) but closely matches the increase for firms with more than 13 establishments (the data and model lines are parallel for large firms). This fit reflects, in part, our calibration targets: the difference in adoption probability between firms with over 600 establishments and those with 13–25 establishments, as well as the aggregate adoption rate of 3%. Because single-establishment firms do not adopt in the model but do so in the data, matching the aggregate adoption rate requires the model to assign higher adoption probabilities to large firms relative to the data.

Panel B plots the cost share of software against the number of establishments for adopting firms. While we target the linear relationships between the cost shares and firm scope in the calibration routine, Figure 8 shows the full relationship across the distribution of firms and compares it to the data. The model captures the magnitude of this relationship well. The biggest firms in the model have a software cost share that is approximately 2.5 percentage points lower than the smallest adopters–closely matching the difference observed in data. Moreover, the model effectively captures the convex relationship between the investment shares and the number of establishments of the firm. This is because, as the firm grows larger, the extra benefit of adding an additional establishment gradually dissipates.

Similarly, Panel C shows that our model is able to match the empirical distribution of the cost share of labor with the number of establishments of the firm.

6 The Aggregate Impact of the Decline in the Software Price

In this section, we use our calibrated model to examine the implications of the productivity improvements in custom software technology over the last 40 years. Note that the model calibration described in the last section calibrates the model to data from the present period. In examining the shock to the software sector, we go back in time, increasing the rental rate of software to its level in the late 1980s while holding all other parameters of the model fixed.

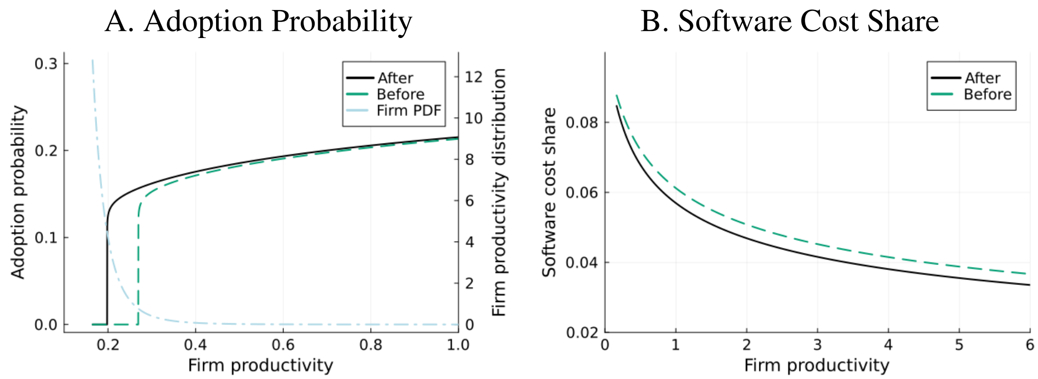
In particular, we feed into the model the change in the productivity of custom software, which maps in to a 63% decrease in the rental rate of custom software between 1987 and 2018. Figure A.3 shows that, while the rental rate of custom software has fallen drastically since the 1980s, the rental rate of other types of capital has remained flat.

6.1 Heterogeneous Impacts

Before discussing the aggregate impacts of the software shock, we first show that the impact is heterogeneous across firms. The heterogeneity is important as it will imply that the impact on aggregate outcomes could be ambiguous.

Software intensity. We start by examining the software shock's impact on software investment intensity. On the extensive margin, Panel A of Figure 9 illustrates the heterogeneous impact of the software shock on firms' software adoption probability. The green dashed and black solid curves

Figure 9: Heterogeneous Impact of Software Shocks on Software Intensity



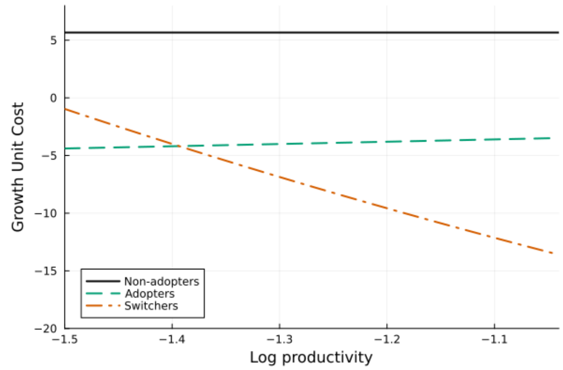
Notes: This figure plots the adoption probability (Panel A) and the software cost share of adopters (Panel B) against firm productivity before and after the software shock.

shows adoption probabilities before and after the shock, respectively. The blue curve plots the productivity distribution (PDF) of firms. Before the shock, only a small share of most productive firms adopt software. The lower software price shifts the productivity threshold of adoption leftward, as the relative gains from adoption increase. This shift causes many firms in the middle range of the productivity distribution, which were previously below the threshold, to adopt software. In addition, adoption probabilities rise even among firms already above the pre-shock threshold, with relatively lower-productivity firms seeing larger increases. For these firms, the profit gain from adoption is greater due to a larger reduction in unit cost as we show below.

On the intensive margin, Panel B shows that, for always-adopters, the software cost share decreases following the shock. This decline reflects the complementarity between software and capital: as software becomes cheaper, firms reduce the cost share on software. Moreover, the relative decline is more pronounced for larger firms.

Unit cost. We now show how the software shock affects the unit cost of production across firms. Figure 10 plots the percentage change in the unit cost from before to after the software shock. It is helpful to separate firms into three categories: firms that never adopt custom software (the black solid line), firms that are always adopters of custom software (the green dashed line), and firms that switch from being non-adopters before the shock to adopters after the shock (the orange dotted line). For firms that never adopt, there is no direct impact from the change in the software price, but their unit cost increases due to the general equilibrium effect on the wage, which increases in response to the overall increase in labor demand from the software shock. The increase in the unit cost for non-adopters is uniform: they all see a rise in their unit cost of about 6 percent regardless of their idiosyncratic productivity. Though it is not shown in this graph, the exit threshold will also

Figure 10: Changes in Unit Cost



Notes: This figure plots the changes in the unit cost after a 63% software productivity decline against the logarithm of firm productivity. The black solid line corresponds to firms that do not adopt software either before or after the shock. The orange dashed line corresponds to firms that switch from non-adopters to adopters when software productivity increases. The green dotted line corresponds to firms that always adopt software.

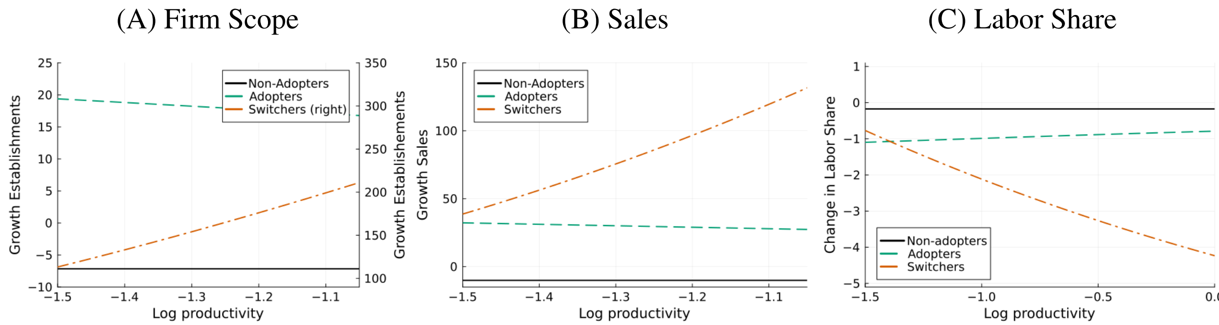
increase because of the wage increase, which lowers the profits of small firms.

For firms that always adopt, the average change in the unit cost is approximately -4%, as the decline in the price of software is offset by the increase in the price of labor. However, the impact is heterogeneous even among adopters, with large adopters experiencing a smaller decrease in their unit cost than the small adopters. This heterogeneity is driven by the non-homotheticity in the production function. Because small adopters devote a larger share of their costs to software, they will see a larger fall in their unit cost in response to the fall in the software price. However, because they also devote a larger share of their costs to labor than bigger firms, they will be impacted more by the general equilibrium rise in wages. On net, the former channel dominates and large adopters experience a smaller decrease in their unit cost than small adopters (the green dotted line is upward sloping).

The orange line shows the percent change in the unit cost for switchers—firms that were non-adopters before the shock and adopters after the shock. For these firms, the growth in the unit cost is downward sloping. This is because the unit cost for adopting firms declines faster with productivity, z, than it does for non-adopting firms because of the increasing returns to scope. So, firms that switch status will have a bigger decline in their unit cost if they are more productive.

Firm scope. Panel A of Figure 11 plots changes in firm scope for always adopters (green dashed line), always non-adopters (black solid line), and switchers (orange dotted line). To improve readability, values for always adopters and always non-adopters are plotted on the left-hand y-axis, while those for switchers are shown on the right-hand y-axis. Always adopters expand their scope while the expansion is decreasing in productivity, mirroring the change in the unit cost. Always

Figure 11: Heterogeneous Impact of Software Shocks



Notes: This figure plots the optimal firm scope (Panel A), firm sales (Panel B), and the cost share of labor (Panel C) against the logarithm of firm productivity before and after a 63% software productivity shock.

non-adopters contract their scope due to higher equilibrium wages. Switchers exhibit the largest increases in scope, capturing the benefit of transitioning from no software to exploiting the increasing returns to scope.

This pronounced expansion among switchers has important implications for establishment concentration. The net effect on the top 1% establishment share depends on where switchers fall in the productivity distribution. Our calibration shows that most switchers occupy the middle productivity range (blue line, Panel A of Figure 9), meaning the top 1% consists primarily of always adopters. This creates competing forces: always adopters gain establishment share from shrinking non-adopters but lose share to rapidly growing switchers. Consequently, the software shock's impact on top 1% establishment concentration remains theoretically ambiguous.

Firm sales. Panel B plots sales growth rates. As with firm scope, switchers and always adopters achieve higher sales due to lower unit costs, with this effect amplified by their expansion of scope. Non-adopters experience declining sales as their unit costs rise. Among always adopters, sales increases are smaller for more productive firms because software represents a smaller cost share for these firms, yielding proportionally smaller unit cost reductions. Switchers experience the largest sales increases. As with firm scope and establishment concentration, these dynamics create offsetting forces, leaving the effect on top 1% sales concentration ambiguous.

Labor share. Panel C shows the response of the labor share. As discussed in Section 5.2, the labor share decreases with firm scope and firm productivity for adopters. Because labor and the capital–software bundle are substitutes, $\sigma_l > 1$, the increase in wages leads to a decline in the labor share for both adopters and non-adopters who substitute away from labor toward the software-capital bundle. For non-adopters, or about 97% of firms in 2018, their labor share decreases by only 0.17 percentage points. The decline in the labor share is larger for adopting firms because adopters further substitute away from labor towards the now-cheaper capital-software bundle as

Table 4: Aggregate Impact of Software Shocks

	1987	2018*	Δ
Aggregate SW investment share			
Model	8.9	10.3	1.4 pps
Data	5.2	10.5	5.3 pps
Share estabs. owned by top 1% of firms			
Model	27.6	29.2	1.6 pps
Data	19.0	27.8	8.8 pps
Sales share by top 1% of firms			
Model	60.4	63.1	2.7 pps
Data	52.5	62.9	10.4 pps
Aggregate labor share			
Model	56.9	56.4	-0.5 pps
Data	62.8	56.5	-6.3 pps
Aggregate TFP			
Model	0.15	0.16	5.83 %
Data	100.0	128.2	28.2 %
Aggregate labor productivity			
Model	0.10	0.11	8.29 %
Data	100.0	163.2	63.2 %

Notes: This table shows how aggregate moments change in response to a 63% decline in the rental rate of custom software. The 2018 moments are targeted in the calibration routine, but the 1987 ones are not.

the rental rate of software falls. The largest decline is seen among the switchers who have now adopted the non-homothetic production function with a declining labor share. Overall, the small change for the median firm and the larger decline in the labor share for larger firms matches the empirical findings from the literature (Autor et al., 2020; Hubmer and Restrepo, 2021; Kehrig and Vincent, 2021).

6.2 Aggregate Impacts

In Table 4, we examine the aggregate implications of the software shock. We emphasize that the moments in 2018 are targeted in the calibration routine described in Section 5.1. However, the change between 1987, the "pre-software" era, and today is untargeted. We are interested in the extent to which the decline in the price of custom software can account for the aggregate trends seen in the data.

Through the lens of the model, the share of firms adopting custom software increases by 2.4 percentage points from just 0.7 percent in 1987. Thus, the shock generates a more than 3-fold increase in the share of firms adopting custom software. Unfortunately, our data on software adoption does not go back to 1987, so we cannot compare this increase to the data. In the data, the aggregate share of investment devoted to custom software increased by 5.3 percentage points. The change in the software price alone generates a 1.4 percentage point increase in the aggregate software investment share or approximately 26% of the data. Later, we consider additional shocks that, through the lens of the model, could have also increased the aggregate software investment share such as a change in the fixed cost of adoption or the weight on software in the production function (a software-biased technical change shock).

Next, we examine the impact of the shock on the increase in establishment and sales concentration. As discussed in Section 6.1, the impact of the software shock on sales and establishment concentration is ambiguous. On the one hand, in response to the shock, adopters increase the average number of establishments they operate while non-adopters decrease their number of establishments, increasing concentration. On the other hand, switchers increase their number of establishments even more than the larger "always adopters." Since most of these switchers are not in the top 1% of firms, this will generate a decline in establishment concentration. On net, the first effect dominates and the share of establishments owned by the top 1% of firms rises by 1.6 percentage points. Thus the model generates 18.2% of the increase in the share of establishments owned by the top 1% of firms.

The model also predicts an increase in sales concentration. Again, the impact of the software shock is ex-ante ambiguous because of the reallocation between the always adopters and the switchers. The sales share of the top 1% of firms increases by 2.7 percentage points in the model or about 26.0% of the increase in the data.

We find that the software price drop leads to a 0.5 percentage point decrease in the aggregate labor share. Compared to the 6.3 percentage points decrease in data, the model accounts for a negligible share of the aggregate decline in the labor share. This is because the decline in the labor share is heterogeneous across firms, with the bulk of firms (the non-adopters) only decreasing their labor share by about 0.17 percentage points.

Finally, we examine how productivity increases in the software-producing sector translate into measured aggregate productivity growth. To do so, we follow BEA methodology and compute aggregate TFP using a Cobb–Douglas production function,

$$Y = ZK^{1-\alpha}L^{\alpha}. \tag{28}$$

Following the BEA, software is treated as part of the capital stock. The aggregate capital stock in the model is constructed by summing traditional and software capital stocks, each denominated in units of final output. While the Solow residual is not the structural measure of technology in our model, it corresponds to the BEA and BLS methodology used to construct empirical TFP measures. It is therefore the appropriate object for comparison with observed TFP growth in the data.

The model generates a TFP growth of 5.83%, 21 percent of the observed growth in data. Several mechanisms contribute to the measured-TFP growth. First, the productivity improvement in the software-producing sector directly lowers the unit cost of production for intermediate good producers. Second, the shock generates reallocation of production toward larger, more productive firms, reflected by increases in sales and establishment concentration. Third, as firms move down the unit cost curve, they benefit from increasing returns to scope, which are also captured in the TFP term within the Cobb-Douglas framework. Fourth, as the rental rate of software declines, it lowers firm output prices and the final good price index. Since fixed and entry costs are paid in final goods, this final good price decline further encourages entry. Additionally, the span-of-control cost—also paid in final goods—declines, further encouraging firm expansion.

To gauge the magnitude of the model–predicted TFP growth, we compare it with the benchmark from Hulten (1978), which states that aggregate productivity growth from a sectoral shock is equal to the product of the sector's sales-to-GDP ratio (i.e., its Domar weight) and its productivity shock: $\frac{\text{Software Investment}}{\text{GDP}} \times \hat{Z}_s$, yielding 2.6% in our case. Our model generates nearly double this effect, primarily because it incorporates monopoly pricing and models the entry and span-of-control as costs paid in the final good, both of which lead to inefficient entry and scope, amplifying aggregate productivity gains.[29]

The last row shows that aggregate labor productivity increases by 8.29%, accounting for about 13 percent of labor productivity growth in the data. Since we normalize aggregate labor supply to 1, this also reflects output growth in the model. The larger increase in labor productivity relative to TFP growth is driven by capital deepening as firms adopt more software and increase their stock of traditional capital, which is complementary with software.

Alternative software shocks. The software productivity shock explains approximately 26% of the 5.3 percentage point increase in aggregate software investment share observed in the data, indicating that additional shocks are necessary to account for the full magnitude of change. In Appendix Section D.5, we calibrate two complimentary shocks alongside the baseline productivity shock to match the complete observed increase. One is a software-biased technical change, modeled as a reduction in the capital's weight relative to software in the production function (γ_k), and the other is a decline in the fixed cost of software adoption ($\bar{F}^S$). These additional shocks amplify the baseline shock's aggregate impacts by 3–4 fold.

[29] Recent literature has extended Hulten's theorem in many ways, considering pre-existing distortions and second-order effects (e.g. Baqaee and Farhi, 2019, 2020; Lashkari et al., 2024).

Table 5: The role of the extensive margin for adoption and firm scope

	Baseline	No Extensive Margin	No Scope
Δ SW investment share	1.41	-1.38	-1.17
Δ Share adopting	2.38	0.0	0.0
Δ Share estabs. top 1%	1.6	-1.2	0.0
Δ Sales share top 1%	2.7	-2.0	-0.2
Δ Aggregate labor share	-0.5	-0.6	-0.3
TFP Growth	5.8	5.7	3.7
Growth labor productivity	8.3	8.7	7.3

Notes: This table reports the effects of the software shock in two alternative versions of the model: (1) "No Extensive Margin", where all firms are adopters; and (2) "No Scope", where all firms are adopters and operate only a single establishment.

6.3 The Role of Adoption and Firm Scope

In this section, we consider two alternative versions of the model in order to understand the roles of the extensive margin choice of adoption and firm scope in determining the aggregate results. First, we consider a version of the model in which all firms are adopters of the software technology. Firms still differ in their scope, and larger firms take advantage of the non-rivalry of software by sharing its cost across their many establishments. Second, we further shut down the choice of firm scope, forcing each firm to operate only one establishment. In this case, non-rivalry is no longer a factor as the software cost cannot be shared across establishments.

In both cases, we recalibrate the model to match the same set of aggregate moments as in the baseline model, with adjustments to account for the altered model structure. Specifically, in the version with no extensive margin, we no longer calibrate the location and scale of the distribution of the fixed adoption cost, $\bar{F}^S$ and ψ. In the version without scope, we no longer calibrate the parameters of the span of control cost, ω_1 and ω_2. Further, without scope, there is no non-homotheticity, so we set the elasticities of substitution σ_k and σ_l to their baseline values. The details of the calibration are given in Table A.14. The sum of squared errors in the calibration is printed in the last row. In each case, the model matches the moments as well as, or better than, the baseline.

We then examine the impact of the software shock within each of the alternative model economies. The results are presented in Table 5. First, row 1 shows that in the absence of the extensive margin, a decline in the price of software would have led to a decrease in the aggregate software investment share. This occurs because software is complementary to capital; as its price falls, firms reallocate investment toward traditional capital. Without the extensive margin, there are no new adopters to offset this decline.

In the model without the extensive margin, concentration would also have declined, as shown by the reduction in the share of establishments and sales held by the top 1% of firms. To a first approximation, the effect of a decline in the price of software is proportional to a firm's software cost share, which decreases with firm size due to the non-homotheticity. As a result, smaller firms benefit more from the software shock than larger firms. In our baseline model, this effect is offset by the extensive margin: the smallest non-adopting firms do not directly benefit from the decline in software prices and are instead adversely affected by the general equilibrium increase in wages. Consistent with evidence that much of the change in sales concentration is driven by changes in scope (Hsieh and Rossi-Hansberg (2023); Smith and Ocampo (2020)), the model with no scope exhibits almost no change in sales concentration.

Finally, the last two rows report the changes in TFP growth and labor productivity. Notably, TFP growth is substantially smaller in the version without scope. This is because reductions in firms' unit costs—arising from returns to scope—are reflected as TFP growth. In our growth accounting exercise, we calculate the Solow residual under the assumption of an aggregate Cobb-Douglas production function with constant returns to scale. In this framework, changes in unit costs (that are not driven by changes in factor prices) are attributed to changes in the Solow residual. Nevertheless, labor productivity growth remains similar across the three models, driven by capital deepening in both software and complementary traditional capital.

These results highlight that although software is non-rival, its adoption does not automatically translate into rising concentration or significant TFP gains. The quantitative outcomes instead depend on our cross-sectional evidence showing that adoption and usage of custom software vary systematically with firm scope.

6.4 Robustness to Excludability and Specificity

Our baseline model assumes custom software investment is non-rival and non-excludable within a firm, allowing firms to costlessly reuse the software across their many establishments. We now consider two extensions that relax this assumption. One is partial excludability. For example, under licensing agreements, vendors may charge based on usage, making software partially excludable. Another is the specificity of the software investment. A piece of software written for one establishment might need to be adjusted for another establishment. Table A.8 summarizes the extended model environment.

First, we model excludability in reduced form by assuming vendor pricing increases with the number of establishments using the software: $r^s(N_i) = r^s N_i^{\phi}$, where ϕ governs the degree of excludability. This is a form of non-linear pricing as in Bornstein and Peter (2024). Second, for specificity, we follow Crouzet et al. (2022b) and assume the firm purchases a CES bundle of software used at each of its establishments: $s_i = (\int_{N_i} s_{ie}^{1/(1-\rho)} de)^{1-\rho} = N_i^{1-\rho} s_{ie}$, where the second equality follows from identical establishments. The parameter ρ controls specificity: $\rho = 1$ corresponds to our baseline (non-specific), while $\rho = 0$ means completely establishment-specific software.

The cost minimization problem for adopters (Equation (11) in the main model) becomes

$$\min_{k_{ie}, l_{ie}, s_{ie}} r^k N_i k_{ie} + w N_i l_{ie} + r^s N_i^{1-\rho+\phi} s_{ie}.$$

The key change is the software cost term: $r^s N_i^{1-\rho+\phi} s_{ie}$ versus $r^s s_{ie}$ in the baseline. When there is no specificity ($\rho = 1$) and no excludability ($\phi = 0$), the two models are the same. To maintain increasing returns to scope, we require $\rho > \phi$, and we restrict our analysis to parameter values satisfying this condition.

While specificity and excludability appear similarly in the firm problem, it is important to note that they differ in market clearing conditions. Under specificity without excludability ($\rho < 1, \phi = 0$), vendors must produce $s_i = N_i^{1-\rho} s_{ie}$ units per firm. Under excludability without specificity ($\rho = 1, \phi > 0$), vendors produce only s_{ie} units but earn profits:

$$\pi_i^s = s_i \left(r_s N_i^{\phi} - 1/Z_s \right).$$

As in the baseline model, we assume that $r_s = 1/Z_s$, so that the vendor would make zero profits in the case where there is no excludability ($\phi = 0$) or when N_i is equal to 1.[30] For simplicity, we assume profits are remitted to the representative household and must be accounted for in the final good clearing condition.

Quantitative analysis. We now examine how our findings change under three scenarios: specificity only ($\rho = 0.8, \phi = 0$); excludability only ($\rho = 1, \phi = 0.2$); and both ($\rho = 0.8, \phi = 0.2$).[31] Panel A of Table A.15 shows how steady-state moments vary with ρ and ϕ, holding all other parameters fixed. Intuitively, lower ρ and higher ϕ reduce the returns to scope from software non-rivalry. This leads to fewer adopters and a lower aggregate software investment share. Among adopters, the span of control shrinks, reducing both sales and establishment concentration. Since non-adopters and smaller firms tend to have higher labor shares, the aggregate labor share rises.

Next, we evaluate how specificity and excludability alter the model's response to a decline in software rental rates. For each scenario, we first recalibrate the model to match the same empirical

[30] We note that given the pricing function, it's possible for the vendor to make negative profits if the firm has less than 1 establishment. This will not happen in practice since all adopting firms have $N_i > 1$.

[31] Setting $\rho = \phi$ would fully offset the non-rivalry of software. In this case, no firm would adopt and the software shock would have no effect.

moments, then introduce the software shock.[32] Panel B of Table A.15 shows how key aggregates respond to the software shock. The baseline results (first column) reprint those in Table 4. Across the alternative calibrations, the qualitative patterns are similar, though the quantitative magnitudes differ slightly.

These differences reflect the recalibrated parameters. As ρ and ϕ vary, so do the implied degrees of non-homotheticity and returns to scope, leading to changes in parameters such as the adoption cost, the elasticities of substitution, and the weight on software. Notably, all three alternative scenarios imply a lower mean of log adoption cost ($\bar{F}^S$), leading to larger increases in adoption and investment following the software shock. Despite higher adoption and investment rates, concentration effects are smaller when only specificity or partial excludability is introduced, as weaker returns to scope dampen the competitive advantage from software investment. By contrast, when both specificity and partial excludability are present, the calibration yields a lower within-firm cannibalization parameter (θ), enabling top firms to expand market share without offsetting losses from their own establishments. In this case, the rise in concentration is slightly larger than the baseline. TFP and output growth and the reduction in the labor share are quantitatively similar to the baseline calibration.

7 Conclusion

This paper examines the implications of the growing importance of custom software investments for the increases in concentration and aggregate productivity. Software is different from other types of investment goods because it is non-rival. Once a firm makes an investment in software, it can use it simultaneously across its many establishments, product lines, or brands.

We build a model of heterogeneous firms that incorporates the non-rivalry of software. In the model, firms choose their scope—or the number of establishments—and whether or not to adopt a technology that uses custom software. If they invest in custom software, the cost can be shared across their many establishments, incentivizing the firm to further increase their scope.

Two theoretical results arise endogenously within the model due to the non-rivalry. First, firms that adopt custom software will have increasing returns to scope. For a given productivity, the unit cost declines with the scope of the firm due to the fact that the firm can share the cost of the non-rival input across more and more establishments. Second, for adopting firms, the non-rivalry leads to a non-homothetic production function; the cost shares of capital, software, and labor all

[32] Recalibration of the model is necessary for a meaningful comparison across the different scenarios, as the uncalibrated models differ substantially in key outcomes (e.g., sales concentration varies by up to 5 percentage points). Table A.16 reports the new parameters and model fit; all three calibrations match the data comparably well.

vary with the scope of the firm.

We use a new dataset on firm-level investments in custom software from the U.S. Census to provide empirical support for both theoretical results. First, we document that, on the extensive margin, the likelihood a firm adopts custom software is increasing in the scope of the firm. Second, we show that, for adopters, the cost shares of software and labor are both declining with firm scope, while the cost share of capital increases with firm scope, consistent with the non-homotheticity in the model.

We use our model to examine the implications of a shock to the price of custom software, calibrated to match the observed 63% decline in the rental rate of custom software between 1987 and 2018. In response to the shock, the adoption rate of custom software increases threefold. The shock can account for about 20% of the increase in the software share of aggregate investment, the share of establishments owned by the top 1% of firms, the sales share of the top 1% of firms, and aggregate TFP growth.

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Online Appendix

A Data

A.1 Annual Capital Expenditure Survey

We use the Annual Capital Expenditure Survey (ACES) from 2002 to 2018. The ACES surveys domestic, private, non-farm companies across all sectors on their capitalized expenditures on structures, equipment (including software), and others. Firms with more than 500 employees are automatically sampled into the survey. Smaller firms are randomly selected based on their industry and payroll. We use the ACES provided sampling weights to ensure that our analysis reflects a nationally representative sample of firms.

The ACES asks firms to report only capitalized expenditures. For software, the instructions further state:

> "Report capital expenditures for computer software developed or obtained for internal use during the year. Capitalized computer software expenditures should consist of costs of materials and services directly related to the development or acquisition of software; payroll and payroll-related costs for employees directly associated with software development; and interest costs incurred while developing the software. IMPORTANT: EXCLUDE CAPITAL EXPENDITURES FOR COMPUTER HARDWARE."

Software obtained for "internal use" includes any software that the firm does not intend to sell to the market. For example, Workday, an HR management software developed by Salesforce, would be considered internal use for any firm that purchases Workday for their own needs. It is external use for Salesforce since they developed it with the intention of selling it to the market. Software developed for internal use is supposed to be capitalized onto a firm's balance sheet according to the GAAP guidelines. We discuss the accounting standards below in Section A.2.

The ACES categorizes software into three types, including (1) pre-packaged, which is purchased off-the-shelf and may include the cost of licensing fees and service/maintenance agreements; (2) vendor customized, which externally developed by a third-party, for internal use; and (3) internally developed, which is developed by the firm's employees and may include payroll.

In this paper, we focus on the vendor customized and internally developed software that closely map into the notion of an input that is non-rival and non-excludable within a firm. We exclude prepackaged software for two reasons. First, though prepackaged software is still non-rival, it is often excludable by the vendor. Often a firm has to buy a separate license for each person or establishment using the product. As a result, it does not scale with the scope of the firm like the input in our model that is non-rival and non-excludable within the firm. Second, investments in pre-packaged software are likely underreported in our data. This is due to the accounting guidelines for handling pre-packaged versus customized software. While vendor customized and in-house developed software should be capitalized on the balance sheet and therefore captured by the ACES, there are exceptions for pre-packaged software, which is often expensed. We discuss the accounting guidelines in Section A.2.

A limitation of the ACES data is firm non-response. While the Census provides weights that correct for non-response in the cross section, irregular participation makes the panel component difficult to use. In particular, although firms with more than 500 employees are intended to be sampled annually, in practice their appearance in the data is uneven, limiting the feasibility of panel-based methods such as the perpetual inventory method.[33] As a result, rather than attempting to recover software capital stocks, we work directly with investment flows and consider alternative assumptions when constructing software cost shares.

Sample selection. To build our sample, we exclude observations that lack data or have negative or zero values for payroll, employment, or sales. We drop observations with missing total capital expenditures, structures investment, equipment investment, or software expenditures, including total software expenditure (T_SOFT), pre-packaged software (P_SOFT), vendor-customized software (C_SOFT), and internally developed software (O_SOFT). We also drop firms who report negative assets at the end of the year and firms whose total software investment is larger than their equipment investment (the software should be included in their value for equipment). Lastly, we apply winsorization to the following variables at the 99.5th percentile within their respective six-digit NAICS industry: total capital expenditures, structure investment, equipment investment, each type of software investment, and custom software per employee.

We merge the ACES data to the Revenue Enhanced Longitudinal Business Database (LBD). After merging, our sample includes 384,000 observations.[34] Table A.1 shows that our sample captures 60-70% of the total capital expenditures in the ACES data. Figure A.2 shows that software investment in the ACES is strongly correlated with software investments from the BEA across sectors.

[33]In practice, restricting attention to firms observed for three to five consecutive years yields a sample with an average firm size more than 100 times that of the average firm in the LBD.

[34]Observation counts are rounded to the nearest thousand in accordance with Census's disclosure review policies.

A.2 Accounting Standards

The ACES survey asks firms to report only software investments that are capitalized onto the firm's balance sheet (U.S. Census Bureau, 2022). One concern is that if the bulk of software investments are expensed instead of capitalized, then our measure of software investments will be inaccurate.

In this section, we discuss the relevant accounting standards for whether software investments should be expensed or capitalized. The ACES documentation specifically refers to "Statement of Position 98-1, Accounting for the Costs of Computer Software Developed or Obtained for Internal Use" when instructing firms about what should be reported as capitalized software expenditures (U.S. Census Bureau, 2022). Statement 98-1 (SOP 98-1) outlines which software development costs should be capitalized versus expensed (American Institute of Certified Public Accountants, 1998). It was then superseded by ASC 350-40, which is the relevant GAAP guideline for the treatment of Internal Use Software (Financial Accounting Standards Board, n.d.). Both statements are similar in their guidance for accounting for internal use software.

The GAAP principles treat software developed for internal use as an intangible. Whether a company should expense or capitalize expenditures related to internal-use software development depends on the stage of the software development project. During preliminary planning and exploratory stages, costs should be expensed as incurred. However, during the actual development of the software, costs should be capitalized. Figure A.1 reprints the GAAP guidelines. Expenses related to paying third-party developers or software purchased from third parties should be capitalized, as should all payroll expenses related to the development of internal-use software (PWC, 2021).

To summarize, the GAAP guidelines require firms to capitalize the costs related to the development of internal-use software, whether done in-house or by a third-party vendor. An exception is for pre-packaged software, which can be expensed if it is not material or has a shelf life of under a year. For these reasons, we believe that pre-packaged software in the ACES is likely underreported, particularly for small firms. However, vendor-customized and own-account software should be well-measured as long as firms are following the accounting standards. Small firms, in particular, might make many mistakes in following these guidelines. In Section B.4, we provide a robustness check of our main results using only the sample of publicly traded firms that are required to follow the GAAP guidelines in their statements to the SEC and are subject to audits.

Furthermore, Figure A.2 shows that software investment in the ACES is strongly correlated with software investments from the BEA across sectors. This is despite the fact that the BEA software investment series do not use the ACES for constructing the software investment series. Instead, they use revenue information from the Economic Census and Services Annual Survey for pre-packaged and vendor-customized software, and they use data on the cost of software inputs (developer time) mainly from the BLS Occupational Employment Statistics Survey (U.S. Bureau of Economic Analysis, 2023).

A.3 Rental Rate of Capital

Following Hall and Jorgenson (1967), we derive the rental rate of capital by the non-arbitrage condition that says a firm should be indifferent between the following two options. In the first option, the firm purchases one unit of capital at price p_{t-1} at the beginning of the period. During the period, it rents capital out at rate R_t. At the end of the period, the firm loses δ_t units of capital to depreciation. The firm resells the $(1-\delta_t)$ unit of capital at price p_t. The total profit is $-p_{t-1}+R_t+(1-\delta)p_t$.

The second option is to save p_{t-1} in the bank and earn interest at rate r_t. Total profit is $p_{t-1}r_t$. The non-arbitrage condition states that the two should be equal and implies that the rental rate of capital is

$$R_t = p_{t-1}r_t + \delta_t p_t - (p_t - p_{t-1}). \tag{29}$$

Suppose the corporate income tax rate is τ_t, the rate of investment tax credit is k_t, and the present value of depreciation allowance is z_t. Then, a more comprehensive formula of the rental rate of capital is

$$R_t = [p_{t-1}r_t + \delta_t p_t - (p_t - p_{t-1})]\frac{1-\tau_t z_t - k_t}{1-\tau_t}. \tag{30}$$

We calculate the rental rate of capital for all assets (r^k) and custom software (r^s), respectively, following Equation (30). We obtain the price indices (p_t), depreciation rates (δ_t), and tax parameters ($(1-\tau_t z_t - k_t)/(1-\tau_t)$) from the Bureau of Labor Statistics (BLS). Particularly, the BLS reports these values for each asset category, including vendor-customized, own-account, and pre-packaged software.[35] We normalize the price indices for all assets and custom software by the Consumer Price Index (CPI) from the BLS. We set the risk-free rate of return to 2%.

Figure A.3 plots the rental rate of capital for all assets and custom software, respectively, from 1988 to 2018.

[35] Underlying data for the rental rate of capital for each asset category are downloaded from: https://www.bls.gov/productivity/tables/rental-prices-major-industries.xlsx. Aggregate data for all assets, equipment, and structures are downloaded from: https://www.bls.gov/productivity/tables/total-factor-productivity-capital-details-major-sectors-and-industries.xlsx.

B Robustness of Empirical Results

In this section, we show the robustness of our empirical results to alternative measures of both software intensity and firm scope. Furthermore, our findings remain robust when incorporating firm fixed effects and using a sample of public firms.

B.1 Alternative Measures of Software Investment Intensity

We demonstrate the robustness of our results, especially the findings related to the intensive margin, across various measures of software intensity in Table A.3.

We present the results by estimating the following regression:

$$Y_{it} = \beta \text{FirmScope}_{it} + \alpha_{it}^{\text{age}} + \alpha_{it}^{\text{industry-year}} + \varepsilon_{it}, \tag{31}$$

where Y_{it} is a measure of the software intensity of firm i in year t, and FirmScope_{it} is a measure of firm scope. We include age and industry-year fixed effects to control for heterogeneous trends across cohorts and industries, respectively. We cluster the standard errors at the industry-year level.

In addition to using the investment rate of custom software (column (2)), we employ alternative metrics such as custom software expenditures per employee (column (3)) and the cost share of custom software (columns (4)–(6)). In particular, the cost share is computed according to Equation (2). Total capital input is measured by total fixed assets, the wage bill by total payroll, but we make different assumptions in measuring software input. In column (4), we use software investment as the software input, assuming full depreciation of software in each period. In column (5), we assume a depreciation rate less than one and then deduce the steady-state software stock by dividing the software investment by its depreciation rate.[36] In column (6), we adjust the total wage bill by subtracting the software investment from payroll, recognizing that the software investment may take the form of wages to developers. Depreciation rates and rental rates of custom software and other types of capital are obtained from the BLS at the 4-digit NAICS level.

Table A.3 provides assurance that the negative coefficient on the intensive margin persists even when employing different measures of software intensity.

[36] Implicitly, this is assuming that the firm has the same investment rate every year. Then, the software stock is equal to S_{it}/δ_{jt}, where S_{it} is the custom software investment of firm i in year t and δ_{jt} is the depreciation rate of custom software for industry j.

B.2 Alternative Measures of Firm Scope

We measure firm scope using the number of establishments in our baseline results. We show that our empirical results are robust to alternative measures of firm scope, including employment, sales, and the number of industries.

Table A.4 reports the results. Column (1) shows that on the extensive margin, doubling the number of establishments is associated with a 0.065 percentage point increase in the likelihood of adopting custom software. Given a 3 percent adoption rate, the estimate can be translated into a more than 100% increase in the adoption rate. Columns (2)–(4) report the coefficients when measuring firm scope by the logarithm of firm employment, sales, and the number of industries in which the firm operates. Consistent with our baseline finding, the likelihood of adopting custom software increases with firm scope.

Columns (5)–(8) report the results on the intensive margin, where we measure software intensity by the share of custom software investment out of total capital investment. We focus on firms with positive investment in custom software, so we are examining the intensive margin conditional on adoption. As shown in column (5), the coefficient on the log number of establishments is estimated at -0.052. The negative coefficient suggests that the software investment share decreases as the firm's number of establishments increases. Considering the average investment share of custom software is 0.378, this estimate indicates a 13.8% ($= 0.052/0.378 \times 100\%$) decrease in the investment share as the number of establishments doubles. Columns (6)–(8) show that this negative relationship between firm scope and software intensity on the intensive margin is robust to other measures of firm scope.

B.3 Firm Fixed Effects

We incorporate firm fixed effects into our regressions to account for time-invariant firm heterogeneity. For these regressions, we restrict the sample to firms with more than 500 employees, which are intended to be included in the sampling frame every year.[37] By leveraging over-time variation, we examine how firms' software investment changes in response to expansions in their scope. We note that the interpretation of these results is different from our baseline findings. Our baseline results show that, in the cross-section, firms with a larger scope are more likely to adopt but have a lower software investment intensity. Here, we show that, for a given firm, as they increase their scope, they are more likely to become adopters and they lower their software investment intensity.

In Table A.5, columns (1) and (3) present our baseline results without firm fixed effects. Col-

[37] In practice, however, even among these larger firms there is substantial non-response.

umn (2) introduces the firm fixed effect, showing that, on the extensive margin, firms are more likely to adopt custom software as their number of establishments increase. Columns (4)-(6) report the intensive margin results using the custom software investment rate, the logarithm of custom software investment per employee, and the cost share of custom software. Consistent with our baseline findings, custom software intensity shows a negative correlation with firm scope. Although the standard errors for the coefficient in the software investment share regression increase, the sign of the coefficient remains negative. The coefficients for regressions of software expenditures per employee and cost share are both negative and statistically significant at the 1 percent level.

B.4 Public Firm Sample

One concern is that firms might not accurately record and capitalize their custom software investments. To address this concern, we merge our ACES sample with the Compustat–SSEL bridge provided by the Census Bureau, resulting in a sample comprising only public firms. Public firms, generally larger in scale, are subject to audit of their financial statements, thus alleviating potential concerns about measurement errors.

This public-firm sample includes 40,000 firm-year observations. Around 40% of them report positive investment in custom software. We repeat the regression analysis in Appendix B.2. Table A.6 shows that the intensive and extensive margin relationship between software investment and firm scope persists when we focus exclusively on public firms.

C Model Appendix

C.1 Firm's Problem

Given productivity z_i, a firm chooses whether to adopt custom software, its firm scope, the price and quantity of each establishment's variety, and factor inputs (including labor, capital, and software if a firm opts for adoption) at each establishment to maximize its net profit given by

$$\Pi(z_i) = \max\{\Pi^A(z_i) - F_i^S, \Pi^{NA}(z_i)\}, \forall z_i,$$

where the superscripts denote non-adoption (NA) and adoption (A) and F_i^S denotes the firm's draw of the fixed-cost of adopting software. Profits are, in turn, given by

$$\Pi^A(z_i) = \max_{p_{ie},y_{ie},N_i,l_{ie},k_{ie},s_{ie}} p_{ie}y_{ie}N_i - (wl_{ie}N_i + r^k k_{ie}N_i + r^s s_{ie}) - F^N(N_i) - F^C, \text{ and}$$
$$\Pi^{NA}(z_i) = \max_{p_{ie},y_{ie},N_i,l_{ie},k_{ie}} p_{ie}y_{ie}N_i - (wl_{ie}N_i + r^k k_{ie}N_i) - F^N(N_i) - F^C.$$

We can solve the profit maximization problem for adopters and non-adopters, respectively, in two steps. We first solve for the firm's cost minimization problem to derive the unit cost of production, and then we solve for the profit maximization problem.

Cost minimization problem. Given the firm's software adoption choice and firm scope, the firm chooses labor, capital, and software (if the firm adopts software) of each establishment to minimize its total production costs. Particularly, for adopters, the firm's cost minimization problem is

$$\min_{l_{ie},k_{ie},s_{ie}} wl_{ie}N_i + r^k k_{ie}N_i + r^s s_{ie}$$
$$\text{s.t.} \quad y_{ie} \le z_i \left[(1-\gamma_l)^{\frac{1}{\sigma_l}} X_{ie}^{\frac{\sigma_l-1}{\sigma_l}} + \gamma_l^{\frac{1}{\sigma_l}} l_{ie}^{\frac{\sigma_l-1}{\sigma_l}} \right]^{\frac{\sigma_l}{\sigma_l-1}}, \text{ where } X_{ie} = \left(\gamma_k^{\frac{1}{\sigma_k}} k_{ie}^{\frac{\sigma_k-1}{\sigma_k}} + (1-\gamma_k)^{\frac{1}{\sigma_k}} s_{ie}^{\frac{\sigma_k-1}{\sigma_k}} \right)^{\frac{\sigma_k}{\sigma_k-1}}, \forall e$$

Because software is non-rival across the firm's establishments, the firm's total software input is the same as the establishment's, i.e., $s_i = s_{ie}, \forall e$.

The first-order conditions w.r.t to l_{ie}, k_{ie}, and s_{ie}, respectively, are

$$l_{ie}: \; wN_i = N_i\lambda_{ie} z_i^{\frac{\sigma_l-1}{\sigma_l}} y_{ie}^{\frac{1}{\sigma_l}} \gamma_l^{\frac{1}{\sigma_l}} l_{ie}^{-\frac{1}{\sigma_l}} \tag{32}$$
$$k_{ie}: \; r^k N_i = N_i\lambda_{ie} z_i^{\frac{\sigma_l-1}{\sigma_l}} y_{ie}^{\frac{1}{\sigma_l}} (1-\gamma_l)^{\frac{1}{\sigma_l}} \gamma_k^{\frac{1}{\sigma_k}} X_{ie}^{\frac{1}{\sigma_k}-\frac{1}{\sigma_l}} k_{ie}^{-\frac{1}{\sigma_k}} \tag{33}$$
$$s_{ie}: \; r^s = N_i\lambda_{ie} z_i^{\frac{\sigma_l-1}{\sigma_l}} y_{ie}^{\frac{1}{\sigma_l}} (1-\gamma_l)^{\frac{1}{\sigma_l}} (1-\gamma_k)^{\frac{1}{\sigma_k}} X_{ie}^{\frac{1}{\sigma_k}-\frac{1}{\sigma_l}} s_{ie}^{-\frac{1}{\sigma_k}}, \tag{34}$$

where λ_{ie} is the Lagragian multiplier.

By Equation (33) and (34), we can write k_{ie} as a function of s_{ie}:

$$k_{ie} = \frac{\gamma_k}{1-\gamma_k} \left(\frac{r^s}{r^k N_i} \right)^{\sigma_k} s_{ie}. \tag{35}$$

Plug this equation into the expression for X_{ie}, and we can write X_{ie} as a function of s_{ie}:

$$X_{ie} = \left[\gamma_k \left(\frac{1}{1-\gamma_k} \right)^{\frac{\sigma_k - 1}{\sigma_k}} \left(\frac{r^s}{r^k N_i} \right)^{\sigma_k - 1} + (1-\gamma_k)^{\frac{1}{\sigma_k}} \right]^{\frac{\sigma_k}{\sigma_k - 1}} s_{ie}. \tag{36}$$

To ease the calculation, denote the unit cost of X_{ie} as p_X. By Equations (35) and (36), we have that

$$p_X \equiv \frac{r^k k_{ie} N_i + r^s s_{ie}}{N_i X_{ie}} = \left[\gamma_k (r^k)^{1-\sigma_k} + (1-\gamma_k) \left(\frac{r^s}{N_i} \right)^{1-\sigma_k} \right]^{\frac{1}{1-\sigma_k}}, \tag{37}$$

which can simplify the expression of X_{ie} to

$$X_{ie} = \frac{1}{1-\gamma_k} \left(\frac{r^s}{N_i} \right)^{\sigma_k} (p_X)^{-\sigma_k} s_{ie}. \tag{38}$$

Now, by Equations (32) and (34), we can write l_{ie} as a function of s_{ie}:

$$l_{ie} = \frac{\gamma_l}{1-\gamma_l} \left(\frac{1}{1-\gamma_k} \right)^{\frac{\sigma_l}{\sigma_k}} X_{ie}^{1-\frac{\sigma_l}{\sigma_k}} \left(\frac{r^s}{w N_i} \right)^{\sigma_l} s_{ie}^{\frac{\sigma_l}{\sigma_k}} = \frac{\gamma_l}{1-\gamma_l} \frac{1}{1-\gamma_k} \left(\frac{1}{w} \right)^{\sigma_l} \left(\frac{r^s}{N_i} \right)^{\sigma_k} (p_X)^{\sigma_l - \sigma_k} s_{ie}, \tag{39}$$

where the second equality follows from Equation (38).

By the expressions in Equations (38) and (39), we can get that the unit cost of production is

$$C_i^A = \frac{w l_{ie} N_i + r^k k_{ie} N_i + r^s s_{ie}}{N_i y_{ie}} = \frac{w l_{ie} + p_X X_{ie}}{y_{ie}} = \left[\gamma_l w^{1-\sigma_l} + (1-\gamma_l)(p_X)^{1-\sigma_l} \right]^{\frac{1}{\sigma_l}}, \tag{40}$$

where p_X is given by Equation (37).

Similarly, we can derive that the unit cost for non-adopters is

$$C_i^{NA} = \left[\gamma_l w^{1-\sigma_l} + (1-\gamma_l)(r^k)^{1-\sigma_l} \right]^{\frac{1}{\sigma_l}}. \tag{41}$$

Profit maximization problem. With the unit cost of production in hand, we can plug it into the firm's profit maximization problem. For adopters, the problem is

$$\max_{p_{ie}, y_{ie}, N_i} N_i p_{ie} y_{ie} - N_i C_i^A(z_i, N_i) y_{ie} - F^N(N_i) - F^C,$$

where the unit cost is given by Equation (40).

Denote the profit of an establishment by

$$\pi_{ie}^{A}(z_i, p_{ie}, N_i) = p_{ie}y_{ie} - C_i^{A}(z_i, N_i)y_{ie} = N_i^{\frac{\theta-\varepsilon}{1-\theta}} P^{\varepsilon} Q(p_{ie}^{1-\varepsilon} - p_{ie}^{-\varepsilon} C_i^{A}(z_i, N_i)), \tag{42}$$

where the second equality follows the demand function facing each establishment $y_{ie} = N_i^{\frac{\theta-\varepsilon}{1-\theta}} p_{ie}^{-\varepsilon} P^{\varepsilon} Q$.[38]

Then, we can simplify the firm's problem to

$$\max_{p_{ie}, N_i} N_i \pi_{ie}^{A}(z_i, p_{ie}, N_i) - F^{N}(N_i) - F^{C}.$$

The first-order conditions w.r.t p_{ie} gives us that the price for each establishment's output is a constant markup over the establishment's marginal cost, which is the same as the unit cost:

$$p_{ie} = \frac{\varepsilon}{\varepsilon - 1} C_i^{A}. \tag{43}$$

Substitute the pricing rule into the establishment's profit in Equation (42), and we have that

$$\pi_{ie}^{A}(z_i, N_i) = \frac{(\varepsilon-1)^{\varepsilon-1}}{\varepsilon^{\varepsilon}} P^{\varepsilon} Q N_i^{\frac{\theta-\varepsilon}{1-\theta}} (C_i^{A})^{1-\varepsilon}. \tag{44}$$

The first-order condition w.r.t N_i yields that

$$\pi_{ie}^{A}\left[1 - \frac{\theta-\varepsilon}{\theta-1} + (\varepsilon-1)\mu_i^{A}\right] = \frac{\partial F^{N}(N_i)}{\partial N_i}, \text{ where } \mu_i^{A} = \frac{(1-\gamma_l)(1-\gamma_k)\left(\frac{r^s}{N_i}\right)^{1-\sigma_k} (p_X)^{\sigma_k-\sigma_l}}{(1-\gamma_l)(p_X)^{1-\sigma_l} + \gamma_l w^{1-\sigma_l}}, \tag{45}$$

is the cost share of software, which determines the firm's optimal scope.

The firm's problem for non-adopters mirrors that for adopters, with $\mu_i^{NA} = 0$ in the first-order condition for N_i.

[38] By the demand curve in Equation (6) and the assumption that all establishments of the firm are symmetric, we can write the demand facing each establishment as $y_{ie} = N_i^{\frac{\theta-\varepsilon}{1-\theta}} p_{ie}^{-\varepsilon} P^{\varepsilon} Q$, where P and Q are ideal price index and aggregate demand, respectively.

D Model Calibration and Identification

D.1 Construction of Data Moments

Table A.9 provides an overview of the data moments along with their respective sources. Here, we provide details on how we construct each data moment. The first set of data moments are calculated using the micro data from our ACES sample. We get the cross-sectional relationship between custom software cost share and the number of establishments by estimating Equation (3). The associated coefficient is reported in Table 2, column (1). Similarly, the cross-sectional relationship for labor share is reported in Table 2, column (2). The share of firms adopting custom software, the average number of establishments and employees per firm are reported in Table 1.

The second set of data moments, including the share of establishments and sales by top 1% firms, is constructed from the U.S. Censuses of Manufacturers, Retail Trade, Wholesale Trade, Services, Utilities and Transportation, and Construction. To consistently calculate these shares in 1987 and 2017, we drop the Census of Finance which starts in 1992. We compute the shares for each 3-digit NAICS industry and then take averages across industries, weighted by the industry's sales share (for the sales concentration) and firm count share (for the establishment concentration).

The third set of data moments is constructed from publicly available data sources. We use the Business Dynamics Statistics (BDS) to calculate the exit rate of age 1 firms. The aggregate labor share is calculated from the Bureau of Labor Statistics (BLS) data, representing the ratio of aggregate labor compensation to value-added output. The aggregate investment share of custom software comes from the Bureau of Economic Analysis (BEA). For consistency with our model, we compute the investment share as the ratio of custom software investment (including vendor-customized and own-account software) to the sum of investment in all software, nonresidential equipment, and nonresidential structures. We exclude investments in other intellectual property categories since these are also non-rival but not explicitly accounted for in our model. All these calculations are based on 2018 data.

Lastly, the Pareto tail for employment comes from Kondo et al. (2023), who estimate the Pareto tail using the Longitudinal Business Database (LBD). We take a midpoint of their estimates using Axtell's method.

D.2 Additional Calibration Results

Figure A.5 shows how the moments respond to a 1 percent increase in each of the parameters, holding all other parameters fixed. From this figure, we can see that most parameters have an impact on many different moments. In this sense, they are all jointly identified. However, there are intuitive links between certain parameters and certain moments.

For example, θ controls the extent to which firms will cannibalize their own sales as they increase their number of establishments. Cannibalization increases with θ, lowering the incentive for firms to expand. As a result, moments related to the skewness of the firm size distribution, such as the sales and establishment share of the top firms, decrease as θ increases. Because the most productive firms do not grow as large, the small firms face less competition, and the exit rate falls.

The parameter α gives the Pareto tail of the underlying productivity distribution. As α increases, so does the Pareto tail of employment in the model. A high α means the underlying productivity distribution has a thinner tail, so moments related to the skewness of employment, sales, and the number of establishments also fall, similar to an increase in θ.

The fixed costs of entry, and production F^E and F^C, and the location and scale of the distribution of fixed adoption costs, $\bar{F}^S$ and ψ, govern the share of firms adopting, average employment and number of establishments, the exit rate, and the relationship between firm size and adoption. The parameter ω_1 governs the importance of the span of control cost while ω_2 governs the elasticity of the span of control cost with the number of establishments. Thus, ω_2 directly impacts many of the measures of skewness in the firm size distribution.

The parameters γ_k and γ_l control the weight on capital, software, and labor in the production functions. Intuitively, the labor share increases with the weight on labor, γ_l, and the average software share of investment decreases with γ_k, the weight on capital as opposed to software. They also have a direct impact on the unit cost and, therefore, the profitability of the firms. As a result, they impact the exit rate since firms exit when their profitability falls below zero.

Finally, σ_l is the elasticity of substitution between labor and the capital-software bundle and σ_k is the elasticity of substitution between capital and software. As discussed in Section 4.4, these will control the cross-sectional relationships between firm scope and the labor, capital, and software shares.

D.3 Elasticities of Substitution Between Factors

The model includes two parameters that govern the elasticity of substitution between input types. σ_k governs the elasticity of substitution between capital and software. σ_l governs the elasticity of substitution between labor and the capital-software bundle. In this section, we discuss how the values of these parameters compare to the previous literature.

In particular, a large literature estimates the elasticity of substitution between capital and labor. Many papers find capital and labor are complements, meaning that it should be less than 1 (Aum and Shin, 2022; Oberfield and Raval, 2021). Others find that they are substitutes, meaning the elasticity should be greater than 1 (Hubmer, 2023; Karabarbounis and Neiman, 2013). Caunedo et al. (2023) and Berlingieri et al. (2022) find that the elasticity of substitution is skill-specific, with low-skill labor being substitutable with capital (an elasticity greater than 1) and high-skill labor being complementary with capital (an elasticity less than 1). Neither of the elasticity of substitution parameters in our model, σ_l or σ_k, is directly comparable to the estimates from this literature. The parameter σ_l will differ from the elasticity of substitution between capital and labor estimated in the literature due to non-homotheticity and the extensive margin choice of whether to use software. Similarly, if one estimated a reduced-form elasticity of substitution between labor and software, it may differ from σ_l, and the reduced-form elasticity of substitution between capital and software may differ from σ_k.

To compare with the estimates in the literature, we calculate the reduced-form elasticity of substitution between capital, labor, and software by shocking the price of capital or labor and resolving the model. For instance, we increase the rental rate of capital by 0.1% or by 10%, resolve the model, and then calculate the elasticity of substitution at the firm level as

$$\sigma_{K,L} = \frac{dln(K_i/L_i)}{dln(w/r^k)}. \tag{46}$$

The formula for the aggregate elasticity of substitution between capital and labor is the same, but using aggregate demand for capital and labor instead of firm-level demand for capital and labor. Analogous formulas give the reduced form elasticity of substitution between capital and software or software and labor.

Table A.10 reports the reduced form elasticities of substitution. Note that the values depend on which price is being shocked, whether one is looking at the aggregate or average elasticity of substitutions, and the size of the price shock. The implied elasticity of substitution between capital and labor (σ_{KL}) is greater than 1, indicating that they are gross substitutes. The elasticity of substitution between software and labor (σ_{SL}) is typically between 1.22 and 1.24. However, in the case of a big shock to wages, the aggregate elasticity of substitution between software and labor falls below 1, driven by firms changing their extensive margin choice of whether to adopt software (when the wage goes up, firms become smaller and firms on the margin become non-adopters who have a higher labor share). To our knowledge, Aum and Shin (2022) is the only paper that estimates the elasticity of substitution between software and labor. Consistent with our estimates, they find an elasticity greater than 1, suggesting that software and labor are substitutes. They also find that software and equipment are complements, consistent with our finding that the elasticity of substitution between software and capital (σ_{SK}) is estimated to be 0.94-0.98.

D.4 Robustness to Alternative Calibrations

This section shows that the main results are robust to two alternative assumptions for the model structure: (1) denominating the fixed costs in units of labor instead of output and (2) an alternative bundling of capital, software, and labor in the CES production function, Eq (9).

Table A.13 presents the details of the calibrations for the alternative models. The model with fixed and entry costs denominated in labor fits the data moments substantially worse than the baseline. Its entry-cost parameter is an order of magnitude larger—necessary to match the average firm size—but because the mass of entrants remains low, only 36% of total labor is devoted to fixed and entry costs (versus 25% of output in the baseline model). As a result, the model implies a labor share above the empirical value, even after reducing the production-function weight on labor, γ_l.

In the model with alternative bundling, the main change to the calibration is in the elasticities of substitution between factors, σ_l and σ_k. In the baseline model, the firm's production function is given by Equation (9). Instead, with alternative bundling, the production function is:

$$y_{ie}^A = z_i \left[\gamma_k^{\frac{1}{\sigma_k}} k_{ie}^{\frac{\sigma_k - 1}{\sigma_k}} + (1-\gamma_k)^{\frac{1}{\sigma_k}} \left(\gamma_l^{\frac{1}{\sigma_l}} l_{ie}^{\frac{\sigma_l - 1}{\sigma_l}} + (1-\gamma_l)^{\frac{1}{\sigma_l}} s_{ie}^{\frac{\sigma_l - 1}{\sigma_l}} \right)^{\frac{\sigma_l}{\sigma_l - 1} \frac{\sigma_k - 1}{\sigma_k}} \right]^{\frac{\sigma_k}{\sigma_k - 1}}, \quad \forall e \in [0, N_i] \tag{47}$$

Here, σ_k is the elasticity of substitution between capital and the software-labor bundle, and σ_l refers to the elasticity of substitution between software and labor. Our calibrated value of $\sigma_k = 0.389$ implies that capital is complementary with labor and software and within the range of the estimates from Oberfield and Raval (2021). The calibrated value of σ_l implies that labor and software are complementary, though the elasticity of substitution is close to 1. Another difference is in the location and scale parameters of the distribution of fixed costs for adoption. Both of these parameters are significantly smaller. In the baseline model, a large scale was required to match non-adoption by the biggest firms; under alternative bundling, the benefit of adoption rises more slowly with firm size. However, we note that in both models, the mode of the fixed cost distribution is close to 0, and firms with the biggest draws of the fixed cost do not adopt. As a result, the share of output devoted to the sum of fixed and entry costs is approximately 25% in both models.

Table A.12 reports the aggregate response to the software shock in the baseline and alternative models. Although the three calibrations differ, they all generate effects of similar size. In the two alternatives, establishment concentration rises by 2.5–3.7 percentage points (versus 1.6 in the baseline) and sales concentration by 4.4 points (versus 2.7 in the baseline). The impacts on TFP growth and on labor productivity are similar in magnitude or larger in both alternative calibrations.

D.5 Alternative Software Shocks

This appendix section investigates two alternative software technology shocks: (i) software-biased technical change, modeled as a reduction in the capital weight relative to software in the production function (γ_k), and (ii) a downward shift in the distribution of the fixed software adoption cost ($\bar{F}^S$).

These two shocks operate through distinct channels. The software weight shock increases the aggregate software investment primarily through intensive margin: as software becomes more important in production, adopters allocate more investment to software, though this has limited impact on adoption rates. In contrast, the fixed cost reduction increases aggregate software investment through the extensive margin by raising the adoption rate.

We calibrate both the software–capital weight and fixed cost shocks so that, together with the baseline software productivity shock, they match the 5.3 percentage point increase in aggregate software investment share in data. As with the baseline, we do so by going back in time: we increase the software price, the weight on capital relative to software, or fixed cost of adoption, while holding other parameters at current "software era" levels. This procedure yields a 12% increase in the software weight relative to capital and a 32% decline in the fixed cost of adoption.

Table A.11 reports the aggregate impact of alternative software shocks. The first two columns reprint the data and baseline software productivity shock effects from Table 4. The third and fourth columns show results from the joint shocks that replicate the 5.3% increase in the aggregate software investment. When software becomes more important (larger weight on software) or more firms adopt software (lower fixed costs), the joint shocks amplify the software productivity shock's effects on concentration, labor share, and aggregate productivity by 3–4 fold.

Interestingly, despite distinct micro-level mechanisms, both shocks generate similar aggregate outcomes. The joint shocks increase the share of establishments owned by top 1% firms by 5.3–6.1 percentage points, explaining 60–70% of the 8.8 percentage points in the data. These shocks slightly overshoot the sales share increase for top 1% firms but produce comparable magnitudes. However, all software shocks generate only moderate declines in aggregate labor share—around 1 percentage point or 20% of the 6.3 percentage points increase in the data. This occurs because labor share decreases among non-adopters (due to wage increases) are minimal, and non-adopters account for most economy-wide labor. Although adopters and switchers experience larger labor share decline, the overall effect remains modest. Finally, the joint shocks increase aggregate TFP by 16 percentage points or 60% of the observed growth, and increase labor productivity by 20.6 percentage points, or 33% of the observed change.

Additional Tables

Table A.1: Share of ACES Public Totals Captured in Our Sample

	Total Software	Pre-packaged Software	Vendor Customized Software	Own Account Software	Total Capitalized Expenditures	Total Equipment	Total Structures
Share	71.06	70.67	68.00	64.90	65.53	66.56	62.61

Notes: For each investment type, the table displays the total in our final ACES sample divided by the publicly released ACES totals, averaged across all years.

Table A.2: Concentration and Software Across Industries

	Estab. Share Top 1%			**Sales Share Top 1%**		
	(1)	(2)	(3)	(4)	(5)	(6)
log(Custom SW)	0.0528***	0.0533***	0.0439***	0.0688***	0.0642***	0.0625***
	(0.0103)	(0.0111)	(0.0111)	(0.0111)	(0.0164)	(0.0171)
log(Pre-Pack. SW)		-0.000839	-0.0132		0.00805	0.00676
		(0.0140)	(0.0141)		(0.0160)	(0.0165)
log(Equip., non SW)			0.0303*			0.0132
			(0.0174)			(0.0159)
log(Structures)			0.00609			-0.00768
			(0.0116)			(0.0126)
log(Employment)	-0.0348**	-0.0345**	-0.0421**	-0.0500***	-0.0529***	-0.0530***
	(0.0153)	(0.0158)	(0.0162)	(0.0158)	(0.0161)	(0.0165)
Constant	0.496***	0.493***	0.415**	0.988***	1.014***	0.981***
	(0.180)	(0.180)	(0.167)	(0.193)	(0.198)	(0.206)
Observations	300	300	300	300	300	300
R-squared	0.195	0.195	0.225	0.313	0.315	0.320
Year FEs	Yes	Yes	Yes	Yes	Yes	Yes

Notes: This table uses industry–year panel data at the 3-digit NAICS level to estimate the relationship between the industry's share of establishments and sales allocated to the largest 1% of firms and custom software investment. Source: ACES, Longitudinal Business Database, and Economic Census.

Table A.3: Software Intensity and Firm Scope: Alternative Measures of Software Intensity

	Extensive Margin	Intensive Margin				
	$\mathbb{1}[I^{sw} > 0]$	Investment Share	Log(Investment Per Worker)	Cost Share	Cost Share Alt.	Cost Share Payroll-adjusted
	(1)	(2)	(3)	(4)	(5)	(6)
$\log(N_{\text{Estab}})$	0.065^{***}	-0.052^{***}	-0.140^{***}	-0.004^{***}	-0.009^{***}	-0.002^{***}
	(0.0016)	(0.0034)	(0.0143)	(0.0007)	(0.0032)	(0.0003)
N	384,000	82,000	82,000	82,000	82,000	82,000
R^2	0.123	0.645	0.635	0.734	0.605	0.733
Ind–Year FE	Y	Y	Y	Y	Y	Y
Age FE	Y	Y	Y	Y	Y	Y

Notes: This table estimates the following regression: $Y_{it} = \beta \text{FirmScope}_{it} + \alpha_{it}^{\text{age}} + \alpha_{it}^{\text{industry-year}} + \varepsilon_{it}$, where Y_{it} is a measure of the software investment of firm i in year t, and FirmScope_{it} is the logarithm of the firm's number of establishments. α_{it}^{age} and $\alpha_{it}^{\text{industry-year}}$ are age and industry-year fixed effects, respectively. The dependent variable for column (1) is an indicator set to 1 if a firm makes positive investment in custom software. The dependent variables for column (2) and (3) are the share of custom software investment out of total capital expenditure, and the logarithm of custom software investment per worker, respectively. Column (4)–(6) use different versions of the custom share of custom software as the dependent variable. (See discussion in Appendix B.1.) Industry is at the 6-digit NAICS level. Standard errors are clustered at the industry-year level.

Table A.4: Software Intensity and Firm Scope: Alternative Measures of Firm Scope

	Extensive Margin: $\mathbb{I}[I^{sw} > 0]$				Intensive Margin: Investment Share			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$\log(N_{\text{Estab}})$	0.065^{***}				-0.052^{***}			
	(0.0016)				(0.0034)			
log(emp)		0.022^{***}				-0.044^{***}		
		(0.0005)				(0.0044)		
log(sales)			0.017^{***}				-0.040^{***}	
			(0.0004)				(0.0038)	
$\log(N_{\text{Ind}})$				0.135^{***}				-0.0892^{***}
				(0.0031)				(0.0061)
N	384,000	384,000	384,000	384,000	82,000	82,000	82,000	82,000
R^2	0.123	0.129	0.127	0.120	0.645	0.652	0.653	0.643
Ind–Year FE	Y	Y	Y	Y	Y	Y	Y	Y
Age FE	Y	Y	Y	Y	Y	Y	Y	Y

Notes: This table estimates the following regression: $Y_{it} = \beta \text{FirmScope}_{it} + \alpha_{it}^{\text{age}} + \alpha_{it}^{\text{industry-year}} + \varepsilon_{it}$, where Y_{it} is a measure of the software investment of firm i in year t, and FirmScope_{it} is a measure of firm scope. α_{it}^{age} and $\alpha_{it}^{\text{industry-year}}$ are age and industry-year fixed effects, respectively. The dependent variable for column (1)–(4) is an indicator set to 1 if a firm makes positive investment in custom software. The dependent variables for column (5)–(8) is the share of custom software investment out of total capital expenditure. Firm scope is measured by the logarithm of the firm's number of establishment in column (1) and (5), the logarithm of employment in column (2) and (6), the logarithm of sales in column (3) and (7), and the logarithm of the number of industries in column (4) and (8). Industry is at the 6-digit NAICS level. Standard errors are clustered at the industry-year level.

Table A.5: Software Intensity and Firm Scope: Including Firm Fixed Effects

	Extensive Margin		Intensive Margin			
	$\mathbb{I}[I^{sw} > 0]$		Investment Share		Log(Investment Per Worker)	Cost Share
	(1)	(2)	(3)	(4)	(5)	(6)
$\log(N_{\text{Estab}})$	0.065***	0.031***	−0.052***	−0.004	−0.134***	−0.001***
	(0.0016)	(0.0056)	(0.0034)	(0.0032)	(0.0262)	(0.0004)
N	384,000	127,000	82,000	49,500	49,500	49,500
R^2	0.123	0.579	0.705	0.753	0.681	
Ind-Year FE	Y	Y	Y	Y	Y	Y
Firm FEs		Y		Y	Y	Y

Notes: This table estimates the following regression: $Y_{it} = \alpha_i + \beta \text{FirmScope}_{it} + \alpha_{it}^{\text{industry-year}} + \varepsilon_{it}$, where Y_{it} is a measure of the software investment of firm i in year t, and FirmScope_{it} is the logarithm of the firm's number of establishments. α_i and $\alpha_{it}^{\text{industry-year}}$ are firm and industry-year fixed effects, respectively. The dependent variable for column (1)–(2) is an indicator set to 1 if a firm makes positive investment in custom software. The dependent variables for column (2)–(4) are the share of custom software investment out of total capital expenditure, the logarithm of custom software investment per worker, and the cost share of custom software, respectively. Column (1) and (3) present the baseline results without firm fixed effects. Industry is at the 6-digit NAICS level. Standard errors are clustered at the industry-year level.

Table A.6: Software Intensity and Firm Scope: Public Firms

	Extensive Margin: $\mathbb{I}[I^{sw} > 0]$				Intensive Margin: Investment Share			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$\log(N_{\text{Estab}})$	0.051***				−0.010***			
	(0.0043)				(0.0024)			
log(emp)		0.069***				−0.011***		
		(0.0050)				(0.0029)		
log(sales)			0.057***				−0.008***	
			(0.0042)				(0.0023)	
$\log(N_{\text{Ind}})$				0.085***				−0.017***
				(0.0085)				(0.0043)
N	40,000	40,000	40,000	40,000	22,000	22,000	22,000	22,000
R^2	0.475	0.489	0.488	0.471	0.610	0.611	0.610	0.610
Ind–Year FE	Y	Y	Y	Y	Y	Y	Y	Y
Age FE	Y	Y	Y	Y	Y	Y	Y	Y

Notes: This table uses the ACES-Compustat merged sample to estimate the following regression:

$$Y_{it} = \beta \text{FirmScope}_{it} + \alpha_{it}^{\text{age}} + \alpha_{it}^{\text{industry-year}} + \varepsilon_{it},$$

where Y_{it} is a measure of the software investment of firm i in year t, and FirmScope_{it} is a measure of firm scope. α_{it}^{age} and $\alpha_{it}^{\text{industry-year}}$ are age and industry-year fixed effects, respectively. The dependent variable for column (1)–(4) is an indicator set to 1 if a firm makes positive investment in custom software. The dependent variables for column (5)–(8) is the share of custom software investment out of total capital expenditure. Firm scope is measured by the logarithm of the firm's number of establishment in column (1) and (5), the logarithm of employment in column (2) and (6), the logarithm of sales in column (3) and (7), and the logarithm of the number of industries in column (4) and (8). Industry is at the 6-digit NAICS level. Standard errors are clustered at the industry-year level.

Table A.7: Cost Shares and Firm Scope: By Establishment Size Categories

	Cost Share of		
	Custom Software	Labor	Capital
	(1)	(2)	(3)
1b.estab_cats×$\mathbb{1}$[SW adopter]	0	-0.0178***	-0.0311***
	(0)	(0.0044)	(0.0048)
2.estab_cats×$\mathbb{1}$[SW adopter]	-0.0171***	-0.0363***	0.0164***
	(0.0034)	(0.0059)	(0.0061)
3.estab_cats×$\mathbb{1}$[SW adopter]	-0.0239***	-0.0334***	0.0198***
	(0.0028)	(0.0067)	(0.0071)
4.estab_cats×$\mathbb{1}$[SW adopter]	-0.0200***	-0.0504***	0.0350***
	(0.0033)	(0.0040)	(0.0042)
5.estab_cats×$\mathbb{1}$[SW adopter]	-0.0211***	-0.0552***	0.0418***
	(0.0023)	(0.0038)	(0.0038)
6.estab_cats×$\mathbb{1}$[SW adopter]	-0.0221***	-0.0534***	0.0401***
	(0.0026)	(0.0035)	(0.0035)
7.estab_cats×$\mathbb{1}$[SW adopter]	-0.0251***	-0.0536***	0.0389***
	(0.0031)	(0.0042)	(0.0042)
1b.estab_cats×$\mathbb{1}$[SW non-adopter]		0	0
		(0)	(0)
2.estab_cats×$\mathbb{1}$[SW non-adopter]		0.00751***	-0.00771***
		(0.0028)	(0.0028)
3.estab_cats×$\mathbb{1}$[SW non-adopter]		0.00256	-0.00309
		(0.0046)	(0.0046)
4.estab_cats×$\mathbb{1}$[SW non-adopter]		0.00782	-0.00809
		(0.0060)	(0.0060)
5.estab_cats×$\mathbb{1}$[SW non-adopter]		0.0114**	-0.0116**
		(0.0052)	(0.0052)
6.estab_cats×$\mathbb{1}$[SW non-adopter]		0.0505**	-0.0504**
		(0.0217)	(0.0217)
7.estab_cats×$\mathbb{1}$[SW non-adopter]		0.0793*	-0.0796*
		(0.0424)	(0.0423)
N	82,000	384,000	384,000
R^2	0.727	0.256	0.256
Ind–Year FE	Y	Y	Y

Notes: This table estimates the following regression:

$$\mu_{ikjt} = \alpha \mathbb{1}[\text{SW adopter}] + \beta_k \mathbb{1}[\text{SW adopter}] + \gamma_k + \delta_{jt} + \varepsilon_{ikjt},$$

where the dependent variable is the cost share of custom software, labor, and capital, respectively, for firm i falling in the k's establishment-size bin. j denotes industry and t year. $\mathbb{1}$[SW adopter] is an indicator set to 1 if a firm makes positive investment in custom software. γ_k is a set of fixed effects for each establishment size bin, and δ_{jt} is industry-year fixed effects. Industry is at the 6-digit NAICS level. Standard errors are clustered at the industry-year level.

Table A.8: The Economic Environment

$Y = \left(\int_{\mathscr{I}} y_i^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}$	Aggregate output
$y_i = \left(\int_0^{N_i} y_{ie}^{\frac{\theta-1}{\theta}} de \right)^{\frac{\theta}{\theta-1}}$	Aggregator of firm varieties
$y_{ie}^{NA} = z_i \left[\gamma_l^{\frac{1}{\sigma_l}} l_{ie}^{\frac{\sigma_l-1}{\sigma_l}} + (1-\gamma_l)^{\frac{1}{\sigma_l}} k_{ie}^{\frac{\sigma_l-1}{\sigma_l}} \right]^{\frac{\sigma_l}{\sigma_l-1}}$	Establishment production for non-adopters
$y_{ie}^{A} = z_i \left[\gamma_l^{\frac{1}{\sigma_l}} l_{ie}^{\frac{\sigma_l-1}{\sigma_l}} + (1-\gamma_l)^{\frac{1}{\sigma_l}} X_{ie}^{\frac{\sigma_l-1}{\sigma_l}} \right]^{\frac{\sigma_l}{\sigma_l-1}}$	Establishment production for adopters
$X_{ie} = \left(\gamma_k^{\frac{1}{\sigma_k}} k_{ie}^{\frac{\sigma_k-1}{\sigma_k}} + (1-\gamma_k)^{\frac{1}{\sigma_k}} s_{ie}^{\frac{\sigma_k-1}{\sigma_k}} \right)^{\frac{\sigma_k}{\sigma_k-1}}$	Capital-software bundle
$\Pi_i^{NA} = \text{sales}_i - wl_i - r^k k_i - F^N(N_i) - F^C$	Firm profit for non-adopters
where $\text{sales}_i = \int_0^{N_i} p_{ie} y_{ie} de$, $l_i = \int_0^{N_i} l_{ie} de$, $k_i = \int_0^{N_i} k_{ie} de$	
$\Pi_i^{A} = \text{sales}_i - wl_i - r^k k_i - r^s(N_i) s_i - F^N(N_i) - F^C$	Firm profit for adopters
where $\text{sales}_i = \int_0^{N_i} p_{ie} y_{ie} de$, $l_i = \int_0^{N_i} l_{ie} de$, $k_i = \int_0^{N_i} k_{ie} de$,	
and $s_i = (\int_0^{N_i} s_{ie}^{\frac{1}{1-\rho}} de)^{1-\rho}$	
and $\mathrm{r}^s(N_i) = r^s N_i^{\phi}$	
$\Pi_i = \max\{\Pi_i^{NA}, \Pi_i^{A} - F^S\}$	Net firm profits
$S = Z_s Y_s$	Software production
$K = Z_k Y_k$	Capital production
$S = \int_{\mathscr{I}} s_i di$	Software resource constraint
$K = \int_{\mathscr{I}} k_i di$	Capital resource constraint
$\Pi^S = \int_0^{N_i} r^s N_i^{\phi} s_i di - Y_s$	Software producer profit
$\Pi^K = \int_0^{N_i} r^k k_i di - Y_k$	Capital producer profit
$L = \int_{\mathscr{I}} l_i di$	Labor resource constraint
$C = wL + \Pi^s + \Pi^k + \int_{\mathscr{I}} \Pi_i di$	Household budget constraint
$Y = C + Y_k + Y_s + F$	Final good resource constraint
where $F = \frac{M\delta F^E}{1-G(z^*)} + M\int_{\mathscr{I}} F^N(N_i) di + MF^C$ $+ MF^S \int_{\mathscr{I}} \mathbb{1}[\Pi_i^A > \Pi_i^{NA}] di$	

Notes: In the benchmark model, we set $\phi = 0$ and $\rho = 1$ so that there is no excludability or specificity. In this case, the software producer makes zero profits. We consider extensions with $0 < \phi < \rho < 1$ in Section 6.4.

Table A.9: Data Moments and Sources

Moment	Data	Source
Cross-sectional relationship between custom software cost share and number of establishments	-0.004	Regression coefficient from Table 2 column (1) using ACES sample
Cross-sectional relationship between labor cost share and number of establishments for adopters	-0.007	Regression coefficient from Table 2 column (2) using ACES sample
Share of adopters	0.03	Table 1 using ACES sample
Avg. number of establishments per firm	1.47	Table 1 using ACES sample
Avg. number of employees per firm	30.7	Table 1 using ACES sample
Share of establishments owned by top 1% firms	0.28	U.S. Censuses*
Sales share of top 1% firms	0.63	U.S. Censuses*
Exit rate of firms with age one	0.21	Business Dynamics Statistics
Aggregate labor share	0.56	Bureau of Labor Statistics
Aggregate investment share of custom software	0.10	Bureau of Economic Analysis
Pareto tail for employment	1.10	Kondo et al. (2023)

Notes: This table summarizes the data moments and sources for calibration. Appendix D.1 provides more details. * We compute the averages of establishments and sales concentration across 3-digit NAICS industries using the Censuses of Manufacturers, Retail Trade, Wholesale Trade, Services, Utilities and Transportation, and Construction. We drop the Census of Finance since it starts in 1992.

Table A.10: Reduced-form Elasticities of Substitution

	Small shock				Big shock			
	Shock w		Shock r		Shock w		Shock r	
	Avg	Agg	Avg	Agg	Avg	Agg	Avg	Agg
σ_{KL}	1.11	1.07	1.12	1.12	1.18	1.11	1.13	1.12
σ_{SK}	–	–	0.96	0.94	–	–	0.96	0.98
σ_{SL}	1.23	1.22	–	–	1.24	0.94	–	–

Notes: Elasticities are undefined if the price of neither factor has been shocked. For example, when shocking the wage, the elasticity of substitution between software and capital is undefined because the denominator of the reduced form elasticity, Equation (46), is indeterminate. The small shock increases the wage or rental rate by 0.1% and the big shock increases the prices by 10%.

Table A.11: Alternative Software Shocks

	ΔData	ΔModel		
		Z_{cs}	$Z_{cs}+\gamma_k$	$Z_{cs}+\bar{F}^S$
Aggregate SW investment share, pps	5.3	1.4	5.3	5.3
Share estabs. owned by top 1% of firms, pps	8.8	1.6	5.3	6.1
Sales share by top 1% of firms, pps	10.4	2.7	10.4	10.5
Aggregate labor share, pps	-6.3	-0.5	-1.3	-1.2
TFP, %	28.2	5.8	16.8	16.5
Labor productivity, %	63.2	8.3	20.7	20.6

Notes: This table shows the aggregate impact of a decrease in the price of software (the baseline shock), plus a decrease in the weight on capital relative to software, and plus a decrease in the fixed cost of software.

Table A.12: Robustness to Alternative Models

	Baseline	F.C. in Labor	Alt. Bundling
Δ SW investment share	1.41	0.87	1.79
Δ Share adopting	2.38	1.81	2.46
Δ Share estabs. top 1%	1.6	3.7	2.5
Δ Sales share top 1%	2.7	4.4	4.4
Δ Aggregate labor share	-0.5	-0.7	-0.4
TFP Growth	5.8	5.3	6.4
Growth Labor Productivity	8.3	9.0	10.5

Notes: This table reports the effects of the software shock in two alternative versions of the model: (1) "F.C. in Labor", where the fixed cost and entry cost are paid in labor; (2)"Alt. Bundling", where software is first combined with labor in an inner CES aggregator, which is then bundled with capital in an outer CES aggregator.

Table A.13: Calibration for Alternative Models

	Values				Moments			
Param.	**Baseline**	**F.C. labor**	**Alt. Bundling**	**Moment**	**Data**	**Baseline**	**F.C. labor**	**Alt. Bundling**
σ_k	0.881	0.911	0.377	Cross-section of sw share	-0.004	-0.004	-0.004	-0.004
σ_l	1.120	1.167	0.953	Cross-section of labor share	-0.007	-0.007	-0.007	-0.007
ω_1	0.06	0.64	0.06	Establishments per firm	1.47	1.45	1.60	1.50
ω_2	1.29	1.39	1.33	Estab share top 1%	0.28	0.29	0.31	0.28
θ	10.89	11.57	12.17	Sales share top 1%	0.63	0.63	0.68	0.63
γ_k	0.63	0.80	0.20	Inv share custom SW	0.10	0.10	0.10	0.10
γ_l	0.76	0.58	0.90	Labor share	0.56	0.56	0.68	0.52
F^E	20.20	319.36	22.78	Employees per firm	30.7	29.8	33.6	30.3
F^C	0.07	1.00	0.08	Exit rate, age 1	0.21	0.21	0.20	0.22
$\bar{F}^S$	32.82	6.03	3.78	Share adopting	0.03	0.03	0.03	0.03
ψ	33.48	37.51	9.37	Adoption 600+ rel. to 13–25	0.34	0.37	0.31	0.34
α	4.75	4.86	4.71	Pareto tail employment	1.10	1.11	1.06	1.12
SSE	—	—	—	—	—	0.01	0.11	0.01

Notes: This table summarizes the model parameters and moments for the model calibration with fixed and entry costs paid in labor and with alternative bundling.

Table A.14: Calibration for Model without Extensive Margin and Scope

	Values				Moments			
Param.	**Baseline**	**No Extensive Margin**	**No Scope**	**Moment**	**Data**	**Baseline**	**No Extensive Margin**	**No Scope**
σ_k	0.881	0.836	0.881	Cross-section of sw share	-0.004	-0.004	-0.004	—
σ_l	1.120	1.235	1.120	Cross-section of labor share	-0.007	-0.007	-0.007	—
ω_1	0.06	0.09	—	Establishments per firm	1.47	1.45	1.47	—
ω_2	1.29	1.27	—	Estab share top 1%	0.28	0.29	0.28	—
θ	10.89	9.13	—	Sales share top 1%*	0.63	0.63	0.62	—
γ_k	0.63	0.83	0.90	Inv share custom SW	0.10	0.10	0.10	0.11
γ_l	0.76	0.75	0.72	Labor share	0.56	0.56	0.57	0.59
F^E	20.20	18.73	31.07	Employees per firm	30.7	29.8	30.6	30.9
F^C	0.07	0.05	0.09	Exit rate, age 1	0.21	0.21	0.21	0.21
$\bar{F}^S$	32.82	—	—	Share adopting	0.03	0.03	—	—
ψ	33.48	—	—	Adoption 600+ rel. to 13–25	0.34	0.37	—	—
α	4.75	4.94	4.90	Pareto tail employment	1.10	1.11	1.06	1.09
SSE	—	—	—	—	—	0.01	0.00	0.01

Notes: This table summarizes the model parameters and moments for the model calibrations without an extensive margin and without scope. With no extensive margin, we no longer calibrate the parameters for the location and scale of the distribution of the fixed adoption cost, $\bar{F}^S$ and ψ. Without scope, we no longer calibrate parameters related to the span of control cost. Further, without scope, there is no non-homotheticity, so we set the elasticities of substitution σ_k and σ_l to their baseline values. *In the model with no scope, there is no cannibalization. Instead, we calibrate the elasticity of substitution between varieties, ε, to match the sales share. We get a value of ε of 5.52.

Table A.15: Robustness to Specificity and Excludability

Panel A: Steady-state moments with different ρ and ϕ, holding all other parameters fixed

	Baseline	$\rho = 0.8$	$\phi = 0.2$	$\rho = 0.8, \phi = 0.2$
SW Investment share	10.3	9.8	9.8	8.5
Share adopting	3.1	2.2	2.2	1.4
Share estabs. top 1%	29.2	26.8	26.8	24.6
Sales share top 1%	63.1	57.7	57.7	52.5
Aggregate labor share	56.4	57.1	57.1	57.7
Aggregate labor productivity	0.1	0.1	0.1	0.1

Panel B: Aggregate impact of software shock with different ρ and ϕ, recalibrating all other parameters

	Baseline	$\rho = 0.8$	$\phi = 0.2$	$\rho = 0.8, \phi = 0.2$
Δ SW investment share	1.4	1.7	1.8	2.1
Δ Share adopting	2.4	2.5	2.6	2.6
Δ Share estabs. top 1%	1.6	1.3	1.3	1.8
Δ Sales share top 1%	2.7	2.6	2.6	2.9
Δ Aggregate labor share	-0.5	-0.5	-0.5	-0.6
TFP Growth	5.8	5.1	5.8	6.9
Growth labor productivity	8.3	7.0	6.9	8.3

Notes: Panel A shows key aggregates under alternative values of ρ and ϕ, holding all other parameters of the model fixed. Panel B shows the change in key aggregates in response to a software shock, under different values of the parameters ρ and ϕ. In the baseline calibration, $\rho = 1$ and $\phi = 0$. In Panel B, the other model parameters are recalibrated for each set of values of ρ and ϕ to match the moments given in Table 3. The details of the calibrations are given in Table A.16

Table A.16: Calibration for Model with Specificity and Excludability

	Values					Moments				
Param.	**Baseline**	**rho=0.8**	**phi=0.2**	**rho=0.8 & phi=0.2**	**Moment**	**Data**	**Baseline**	**rho=0.8**	**phi=0.2**	**rho=0.8 & phi=0.2**
σ_k	0.881	0.845	0.839	0.807	Cross-section of sw share	-0.004	-0.004	-0.004	-0.004	-0.004
σ_l	1.120	1.150	1.147	1.208	Cross-section of labor share	-0.007	-0.007	-0.007	-0.007	-0.007
ω_1	0.06	0.06	0.07	0.06	Establishments per firm	1.47	1.45	1.48	1.37	1.53
ω_2	1.29	1.30	1.31	1.32	Estab share top 1%	0.28	0.29	0.28	0.27	0.28
θ	10.89	10.33	10.28	10.08	Sales share top 1%	0.63	0.63	0.62	0.61	0.62
γ_k	0.63	0.62	0.62	0.67	Inv share custom SW	0.10	0.10	0.11	0.11	0.11
γ_l	0.76	0.77	0.77	0.75	Labor share	0.56	0.56	0.57	0.58	0.56
F^E	20.20	20.03	19.34	19.70	Employees per firm	30.7	29.8	30.5	30.6	29.0
F^C	0.07	0.07	0.07	0.06	Exit rate, age 1	0.21	0.21	0.21	0.21	0.21
$\bar{F}^S$	32.82	26.24	19.94	7.40	Share adopting	0.03	0.03	0.03	0.03	0.03
ψ	33.48	32.58	28.47	18.06	Adoption 600+ rel. to 13–25	0.34	0.37	0.34	0.34	0.30
α	4.75	4.73	4.77	4.69	Pareto tail employment	1.10	1.11	1.11	1.11	1.09
SSE	—	—	—	—	—	—	0.01	0.00	0.01	0.03

Notes: This table summarizes the model parameters and moments for the model calibration with partial excludability and specificity.

Additional Figures

Figure A.1: PWC Handbook on the GAAP Guidelines

ASC 350-40-30-1

Costs of computer software developed or obtained for internal use that shall be capitalized include only the following:

a. External direct costs of materials and services consumed in developing or obtaining internal-use computer software. Examples of those costs include but are not limited to the following:

1. Fees paid to third parties for services provided to develop the software during the application development stage

2. Costs incurred to obtain computer software from third parties

3. Travel expenses incurred by employees in their duties directly associated with developing software.

b. Payroll and payroll-related costs (for example, costs of employee benefits) for employees who are directly associated with and who devote time to the internal-use computer software project, to the extent of the time spent directly on the project. Examples of employee activities include but are not limited to coding and testing during the application development stage.

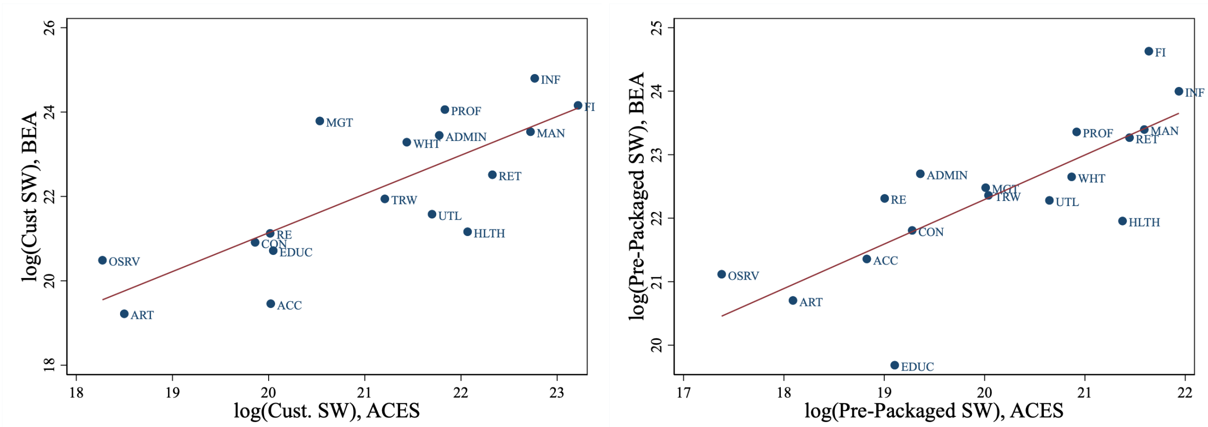
c. Interest costs incurred while developing internal-use computer software. Interest shall be capitalized in accordance with the provisions of Subtopic 835-20.

Source: PWC (2021)

Figure A.2: Software Investment: BEA vs. ACES

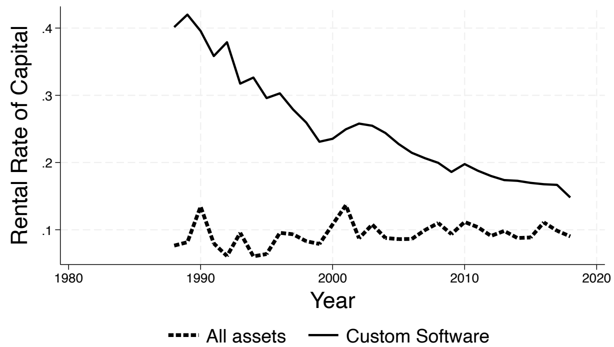
(A) Custom and Own-Account Software

(B) Pre-packaged Software



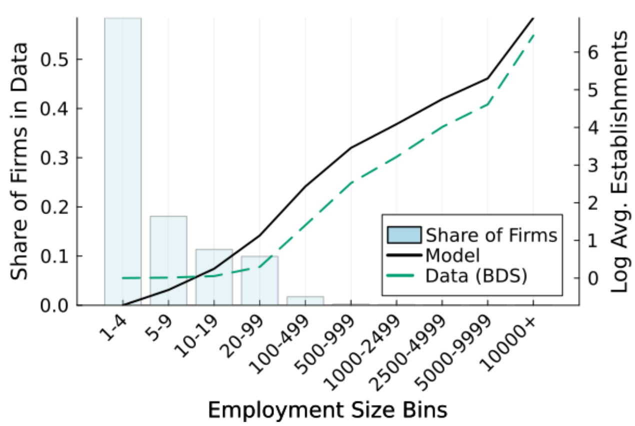
Notes: Figures show the correlation across sectors between custom and own-account software (panel A) and pre-packaged software (panel B) in the BEA versus the ACES. Source: BEA National Accounts Data and ACES.

Figure A.3: Rental Rate of Capital



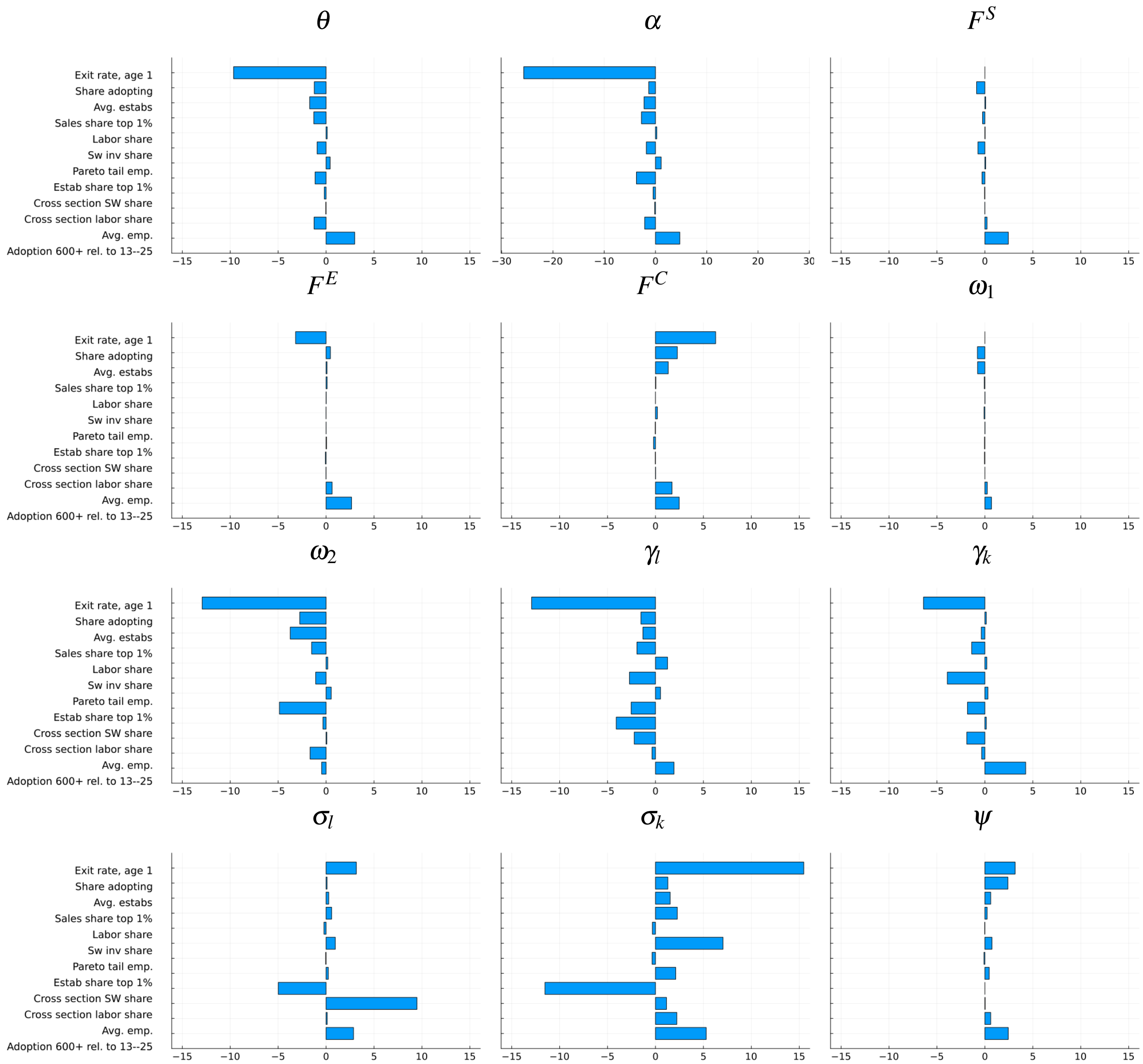
Notes: This figures shows the rental rate of capital (including all types of assets) and that of custom software from 1988 to 2018. Appendix A.3 provides more details on the construction of these rental rates.

Figure A.4: Span-of-control in the Model and Data



Notes: This figure compares the model-implied average number of establishments of firms in each employment size bin to the averages computed from the Business Dynamics Statistics (BDS, green dashed line).

Figure A.5: Response of Moments to a 1% Increase in Parameters



Notes: This figure shows the percent change in each moment in response to a 1% increase in each parameter, holding the other parameters at optimal values.